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Investigating **Newton's Law of Cooling** for Fluids and Granular Solids: A **Regression-Based Model** Fitting Approach

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Abstract

Natural Cooling of a fluid follows the Newtonian cooling mechanism having exponential decay curve. This curve models the cooling process as a **single order exponential decay**.

In this study, cooling rates of fluids like water, coffee, mustard, oil and of granular solids like sand were studied. Temperature sensors interfaced with microcontroller were used for temperature data acquisition. Computational approach includes curve fitting using **Levenberg-Marquardt algorithm** along with data splitting (training and validation sets), and residual error analysis. Residuals are computed and analyzed to assess the quality of the fit. By comparing the training and validation errors, the generalizability of the model is evaluated.

Introduction

According to Newton law of cooling, the rate of cooling of a hot object is directly proportional to the difference in temperature between the hot body and its ambient environment. Mathematically, it is modelled by the differential equation,

$$\frac{dT}{dt} = -k(T - T_{amb})$$

where Tamb is the **ambient temperature**.
The solution of the equation follows the single exponential decaying curve of the form;

$$T(t) = T_0 e^{-kt} + T_{amb}(1 - e^{-kt})$$

where T_0 is the **initial temperature** of the body. Many experimental studies, however, report that the cooling follows a second order exponential decay. Present study investigates the order of exponential decay for fluids and granular solids. .

Methodology

Five different fluids water, coffee, mustard oil engine oil and granular solids like sand was heated to maximum temperature possible in the laboratory conditions (also dependent on material property). Data acquisition of cooling temperatures with time for these materials is done by interfacing the **MAX6675 module** having K-type thermocouple with an **ATmega 328 microcontroller** (Arduino board). The given data set in each case is divided into two parts with 80% of data being used for training the model and the rest 20% for validating it. The data splitting is done using the train test split function in the model selection module of the **scikit-learn** library in Python. The curve-fitting process is executed using the **Levenberg-Marquardt algorithm.**

Observations & Data Analysis

First and second order exponential fit was performed on cooling temperature data set for each fluid was performed and is shown in following figures. Data set is divided into parts with **80 percent** training data set and fitting parameters were obtained. The same fitting parameters were used for validation data set (**20 percent** data set). All the fitting parameters for all fluids and granular solid are shown in table 1 . Residuals were estimated by comparing the true data point and estimated curve and as can be observed, residuals values show a large dispersion in first order fitting in comparison to second order exponential fit and similar results were obtained for all fluids. Fluid which attains high temperature (granular solid) shows a significant difference in residuals values thus embark the validity of second order exponential fit.

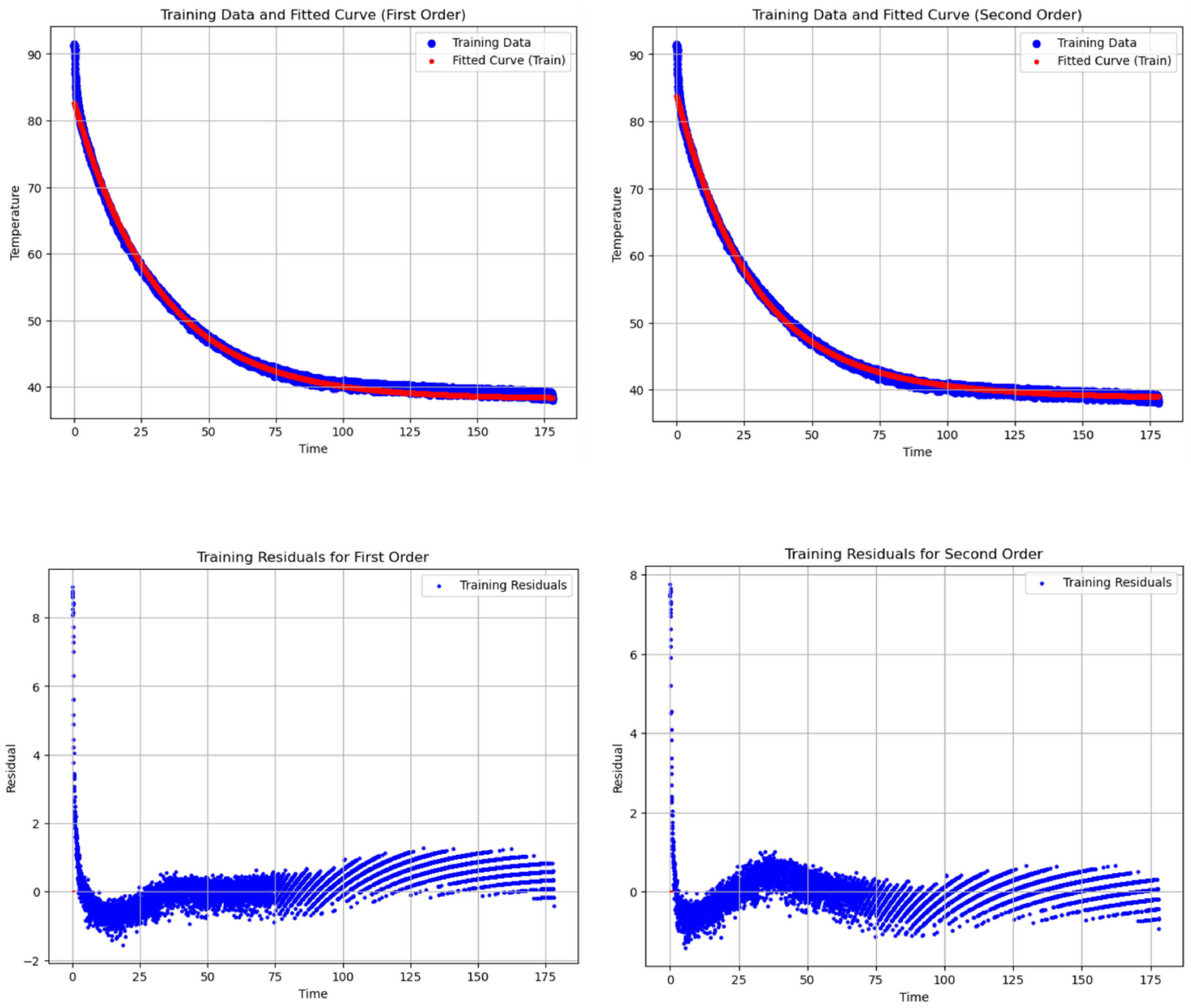
Fluid	Initial Temperature	First order Exponential fit $y = A\,e^{-kt}$	Second order Exponential Fit $y = A_1\,e^{-K_1 t} + A_2\,e^{-K_2 t}$
Water	92°C	$A = 44.4$ $k = 5.3 \times 10^{-4}\,/s$	$A_1 = 42.3,\ k_1 = 6.0 \times 10^{-4}\,/s$ $A_2 = 3.1,\ k_2 = 1.5 \times 10^{-4}\,/s$
Coffee	100°C	$A = 54.0$ $k = 5.5 \times 10^{-4}\,/s$	$A_1 = 14.5,\ k_1 = 1.1 \times 10^{-2}\,/s$ $A_2 = 51.4,\ k_2 = 5.3 \times 10^{-4}\,/s$
Engine oil	150°C	$A = 110.3$ $k = 3.35 \times 10^{-4}\,/s$	$A_1 = 61.0,\ k_1 = 6.2 \times 10^{-4}\,/s$ $A_2 = 56.2,\ k_2 = 2.3 \times 10^{-4}\,/s$
Mustard Oil	170°C	$A = 118.6$ $k = 8.04 \times 10^{-4}\,/s$	$A_1 = 87.7,\ k_1 = 1.1 \times 10^{-3}\,/s$ $A_2 = 35.7,\ k_2 = 4.6 \times 10^{-4}\,/s$
Sand (Granular solid)	270°C	$A = 230.7$ $k = 9.9 \times 10^{-4}\,/s$	$A_1 = 85.1,\ k_1 = 1.5 \times 10^{-3}\,/s$ $A_2 = 150.4,\ k_2 = 8.3 \times 10^{-4}\,/s$

Let's Visualize!

We compare the **effectiveness** of first-order and second-order exponential models in fitting temperature decay over time for the first fluid i.e. water. The first-order exponential fit (top-left plot) captures the general trend but shows slight deviations, particularly at the beginning and end of the cooling process. In contrast, the second-order fit (top-right plot) closely follows the data points, indicating that it can better model the temperature decline with more flexibility due to the inclusion of additional parameters.

The residual plots (bottom row) further highlight the differences in model accuracy. For the first-order fit (bottom-left), the residuals range from **-2 to +9** and display oscillatory patterns, especially in the later stages of cooling. This systematic variation suggests that the first-order model struggles to capture all aspects of the cooling behavior, resulting in recurring errors. On the other hand, the residuals for the second-order fit (bottom-right) are generally closer to zero and vary within a narrower range of **0 to 8**, indicating a more accurate fit.

Henceforth, the second-order exponential model provides a superior fit for Newton's cooling data.



Conclusion

- In refining the model and ensuring the fidelity of the fitted parameters, the respective cooling rates in each case. This methodology offers a **robust framework** for analyzing thermal processes and can be extended to other experimental setups involving heat transfer.

- Our results indicate that a **double exponential decay** model provides a better fit with lower values of residual errors in all cases, thereby, capturing the nuances of the cooling process more accurately. The study also highlights the role of data visualization and residual analysis.

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