National Competition in Computational Physics (NCICP) 2024

Pedagogical Insights and Numerical Simulations of Chladni Patterns Using Spectral Methods.

Affiliation: Hindu College, University of Delhi, New Delhi 110007

Team Members :-

- Seema(Team Leader)
- Ishu
- Vanshika

Supervisors :-

- Prof. Pragati Ashdhir
- Dr. Neha Batra

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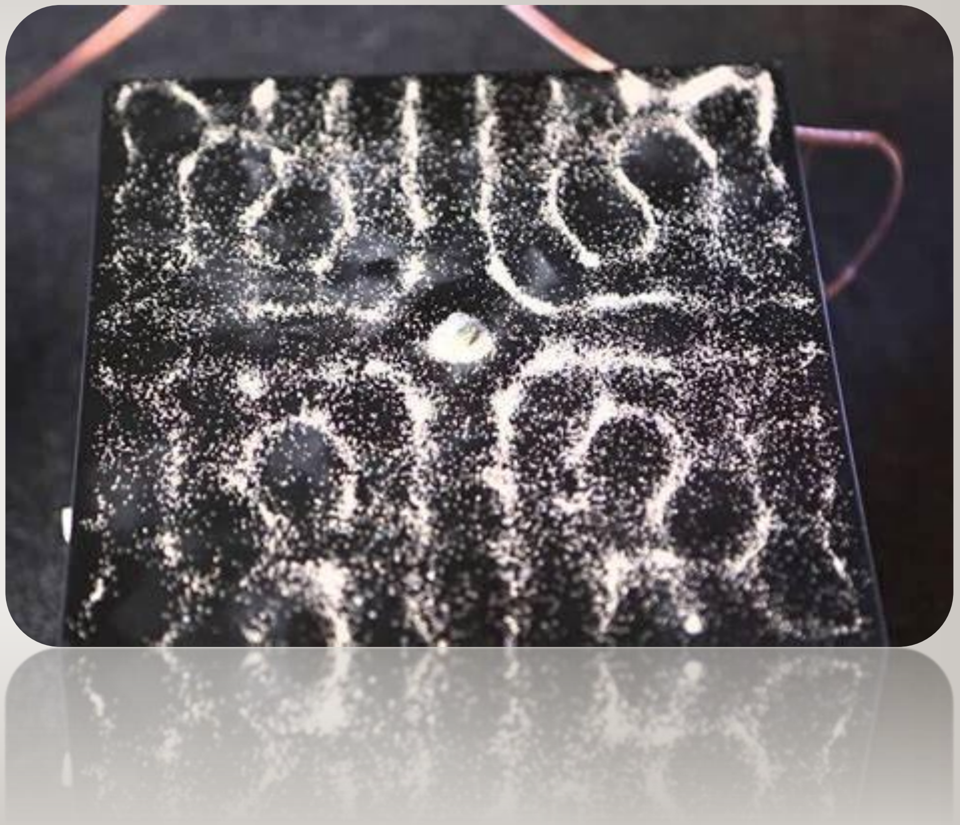
TABLE OF CONTENT

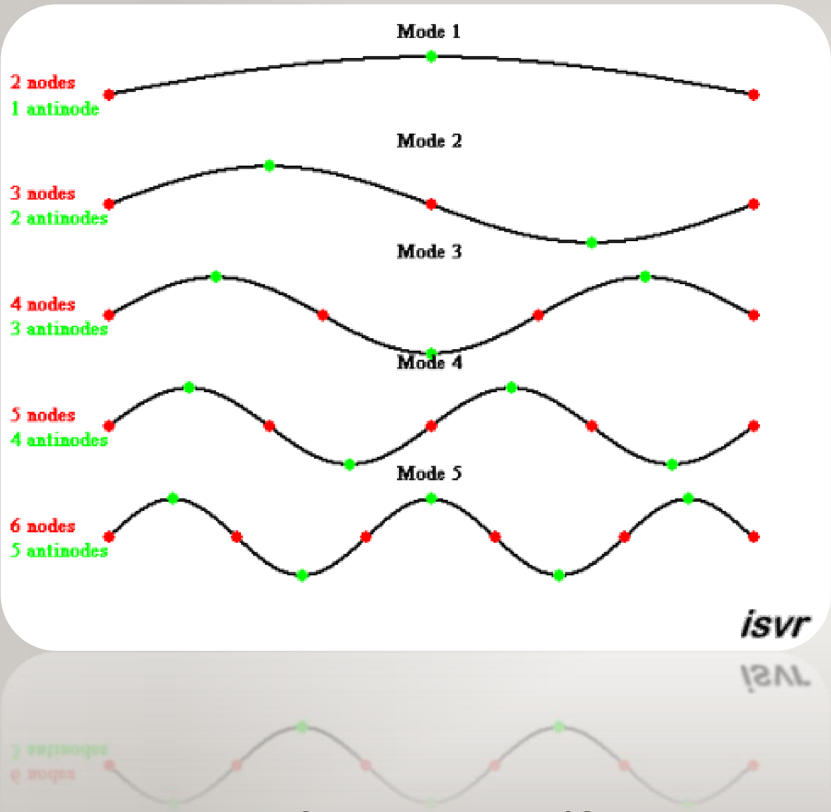
Introduction

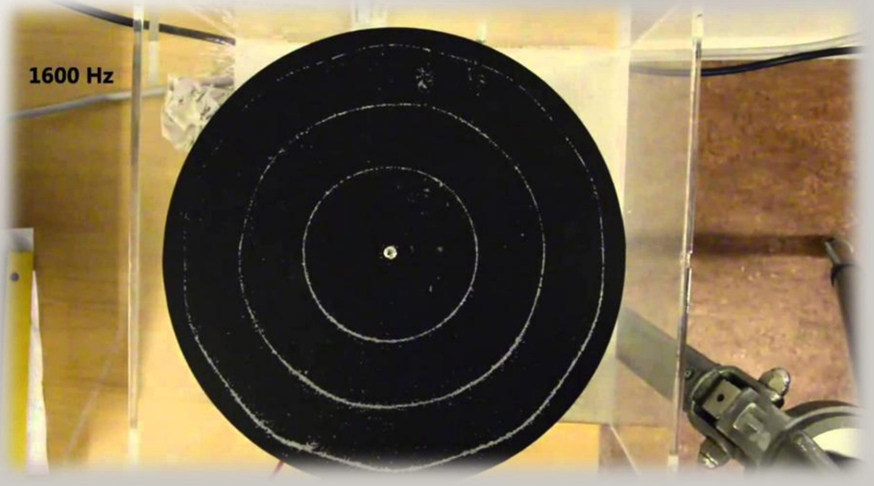
Chladni patterns

Chladni patterns, named after German physicist Ernst Chladni, are visual representations of the **modes of vibration** on a rigid surface. When a surface, such as a metal plate, is vibrated at certain frequencies, sand sprinkled on the surface accumulates along the **nodal lines** where there is no movement, forming intricate and symmetrical patterns.



Modes of vibration and some examples





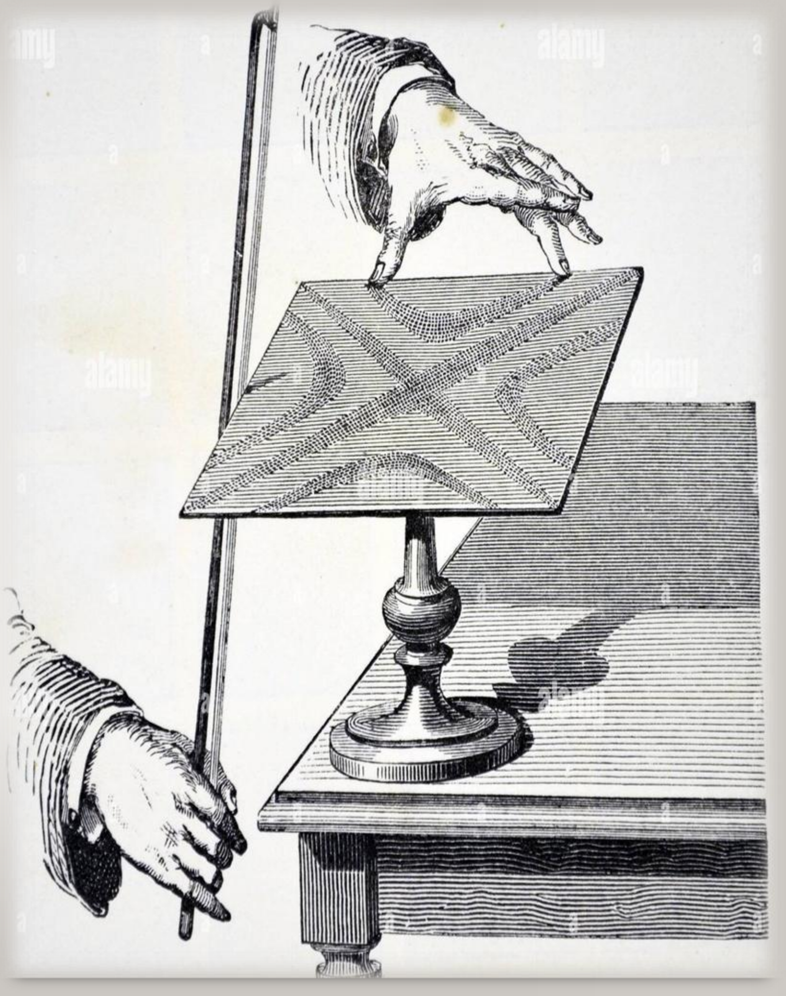
- Vibration of a plate (2D membrane)
- Chladni patterns on circular plate

Background

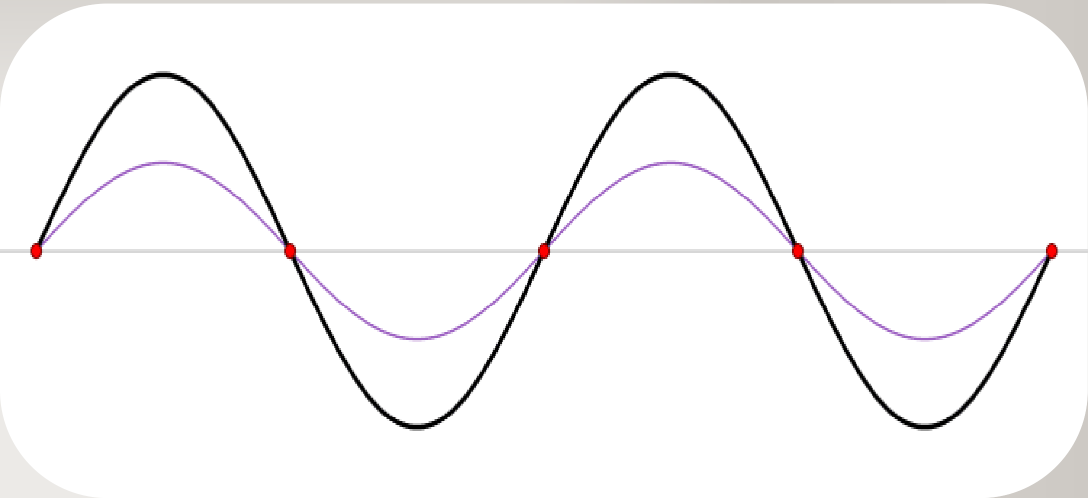
- German physicist and musician Ernst Florens Friedrich Chladni
- Born :- November 30, 1756
- Father of acoustics and meteoritics.
- In 1787 , book Entdeckungen über die Theorie des Klanges ("Discoveries in the Theory of Sound ")

Production of Chladni Pattern

- The shape of a Chladni plate can be square, rectangular, circular, or even shaped like a violin or guitar body, as long as it has a fixed constraint at the center.
- The plate is dusted with a material in order to see the patterns, such as flour, sand, or salt.
- Next, the plate is excited by drawing a violin bow across the side of the plate until it reaches resonance.
- The sand moves away from the antinodes, where the amplitude of the standing wave is maximum, and toward the nodal lines, where the amplitude is minimum, forming patterns known as Chladni figures.



Theoretical Background



Wave and wave theory

Waves are basically **disturbances** in space which transfer energy through space or a medium.
Wave theory explains the phenomena of waves, their behavior, their properties and their propagation in space.

Standing waves

Standing waves, also known as stationary waves, occur when two waves of the same frequency and amplitude travel in opposite directions and interfere with each other. This interference creates a pattern that appears to be standing still, characterized by nodes (points of no displacement) and antinodes (points of maximum displacement).

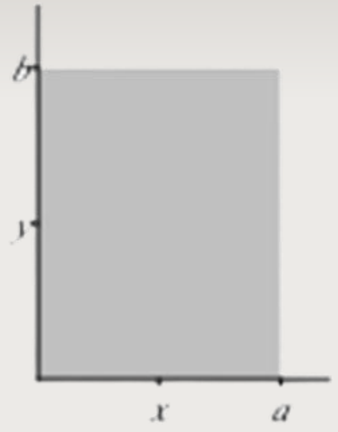
Resonance

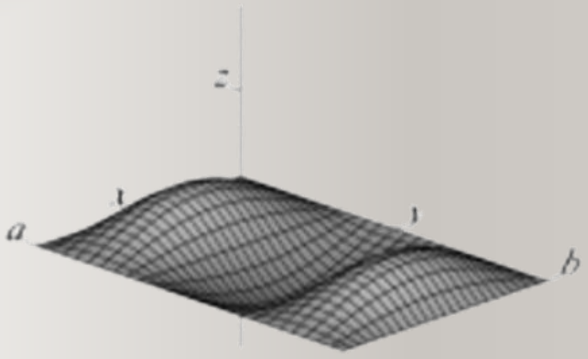
Resonance is the phenomenon where a system or an object is driven at its natural frequency of vibration and then due to superposition principle the amplitude of the vibration is maximized.

Mathematical Formulation

The classical wave equation in 3-D is given by

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$





$$u_{tt} = c^2 \nabla^2 u = c^2 (u_{xx} + u_{yy})$$

$u(x, y, t)$ = deflection of membrane from equilibrium at position (x, y) and time t

For a fixed t, the surface $z = u(x, y, t)$ gives the shape of the membrane at time t

Mathematical Formulation

The analytical solution of a 2-D wave equation for given boundary conditions is:

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \sin(\mu_m x)\sin(v_n y)(B_{mn}\cos(\lambda_{mn}t) + B^*_{mn}\sin(\lambda_{mn}t))$$

Where $\mu_n = \frac{m\pi}{a}, v_n = \frac{n\pi}{b}$, $\lambda_{mn} = c\sqrt{\mu^2{}_m + v_n{}^2}$

$$B_{mn} = \frac{4}{ab}\int_0^a\int_0^b f(x,y)\, sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right) dydx$$

$$B^*_{mn} = \frac{4}{ab\lambda_{mn}}\int_0^a\int_0^b g(x,y)\, sin\left(\frac{m\pi x}{a}\right) sin\left(\frac{n\pi y}{b}\right) dydx$$

Helmholtz Equation: Time independent form

$$\frac{\partial^2 u(r,t)}{\partial t^2} == c^2 \nabla^2 u(r,t)$$

Converting time dependent wave equation to time dependent form using separation of variables method.

Temporal Equation

$$\frac{d^2 T(t)}{dt^2} + \omega^2 T(t) = 0$$

Spatial Equation

$$\nabla^2 \psi(r) + \frac{\omega^2}{c^2} \psi(r) = 0$$

Helmholtz Equation

Spatial Equation

$$\nabla^2\psi(r) + \frac{\omega^2}{c^2}\psi(r) = 0$$

Temporal Equation

$$\frac{d^2T(t)}{dt^2} + \omega^2 T(t) = 0$$

This spatial equation is known as the **Helmholtz equation:**

$$\boldsymbol{\nabla^2\psi(r) + k^2\psi(r) = 0}$$

Where $\boldsymbol{k = \frac{\omega}{c}}$ is the wave number.

Obtaining Chladni Pattern

1. Finite Difference Method
2. Fast Fourier Transform
3. Helmholtz Equation

1 Finite Difference Method

1 Dirichlet Boundary Condition

Pedagogical Insights

Visualization of Wave Phenomena

Interdisciplinary Connections

Active Learning and Experimentation

Critical Thinking and Problem-Solving

- **Conceptual Understanding**

  - Help students to grasp abstract ideas like wave interference , resonance and node formation.

  - Bridges the gap between Mathematical theory and physical reality.

Interpretation of results

Methods used:

- Finite Difference Method (Crank Nichelson Method)
- Spectral Method (Fast Fourier Transform)
- Helmholtz Equation (Eigenvalue Problem)

Boundary Conditions Applied:

- Dirichlet Conditions (Fixed Boundary Conditions)
- Neumann Conditions (Free boundary conditions)

Initial Conditions tested:

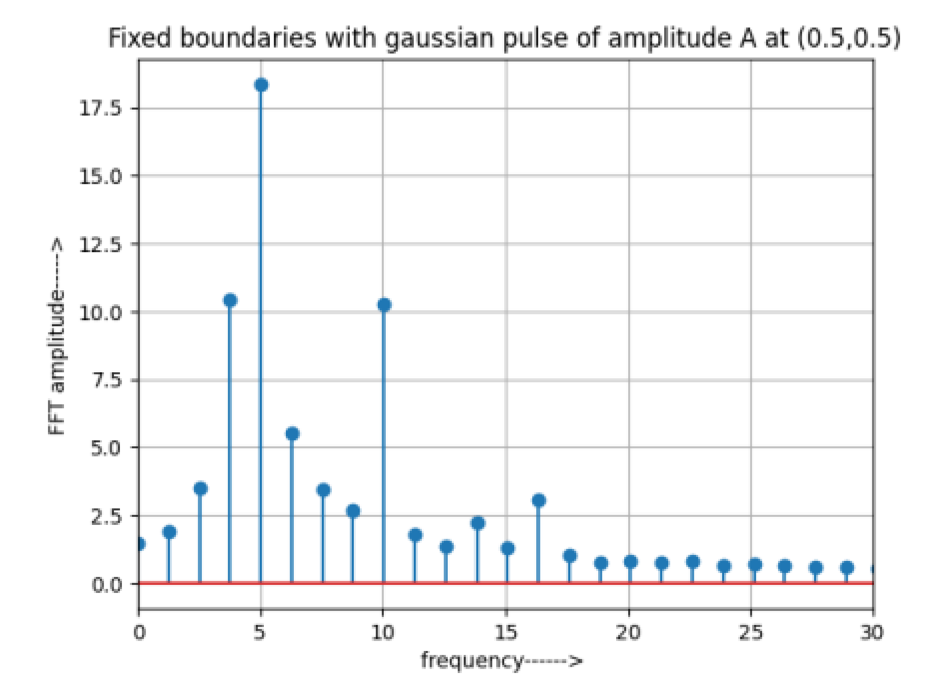
- Gaussian Pulse at the center of amplitude A
- Gaussian Pulse at the center of amplitude 5A
- Circular wavefront at the center
- Square wavefront at the center
- Multiple gaussian function at the center

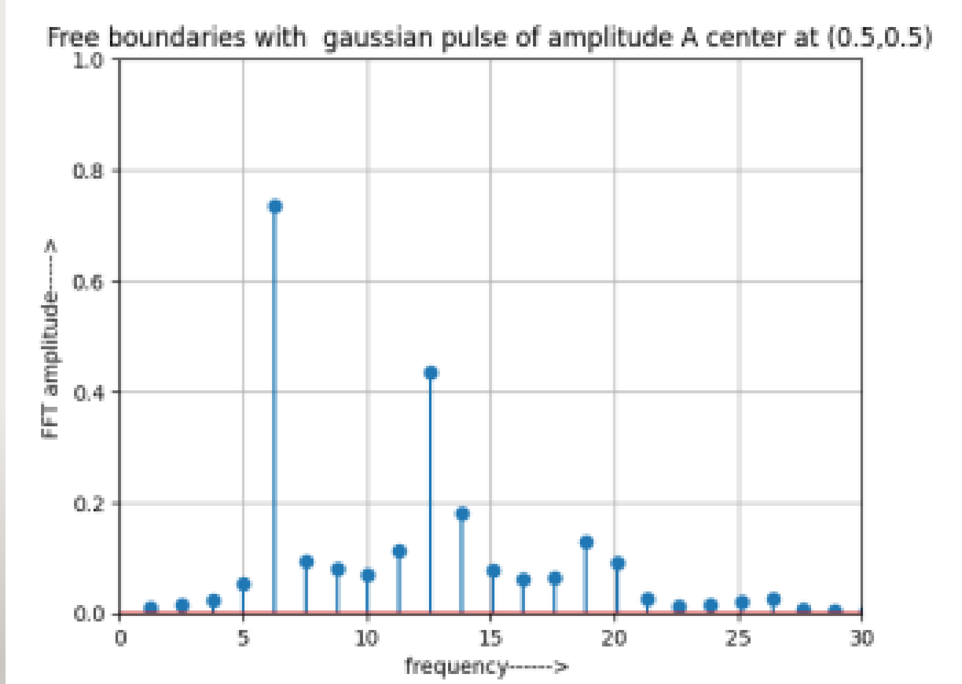
Effect of initial and boundary condition

Free v/s Fixed boundary for same initial condition

Amplitude variation of initial function with same boundary

Same boundary conditions with different initial conditions





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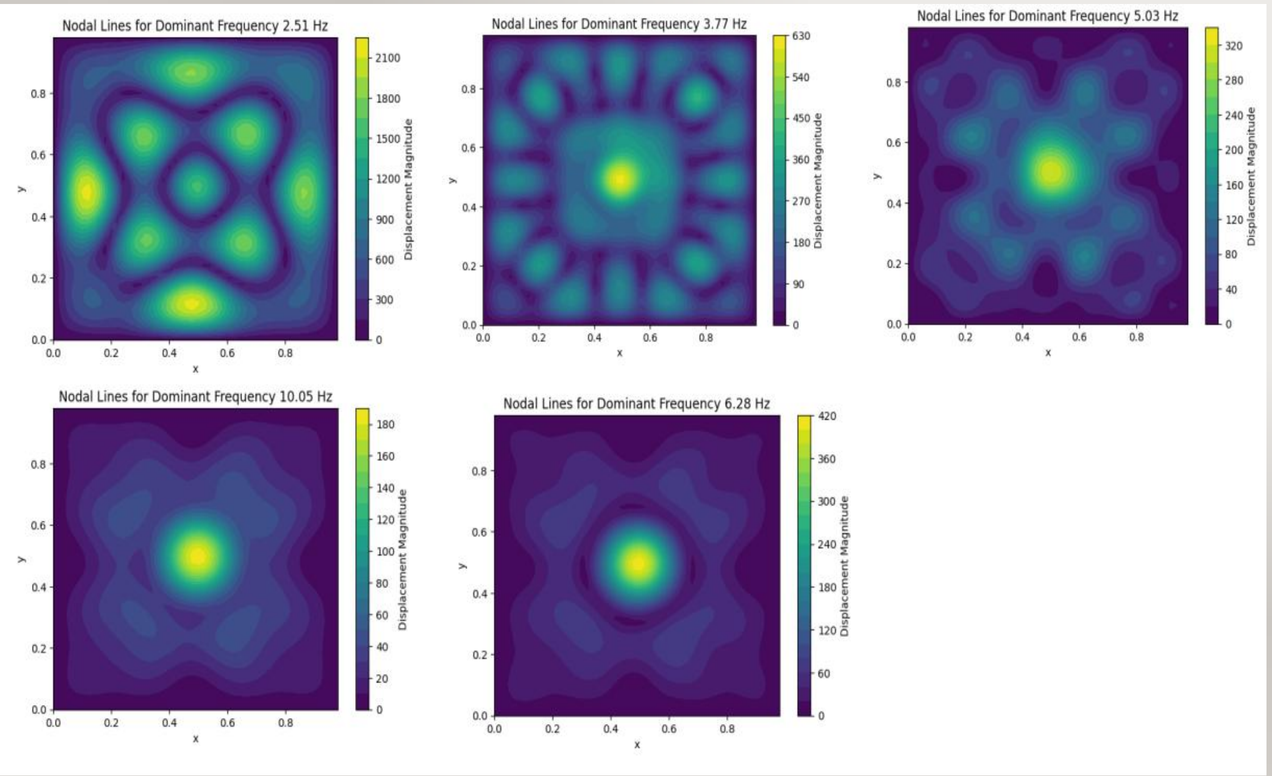

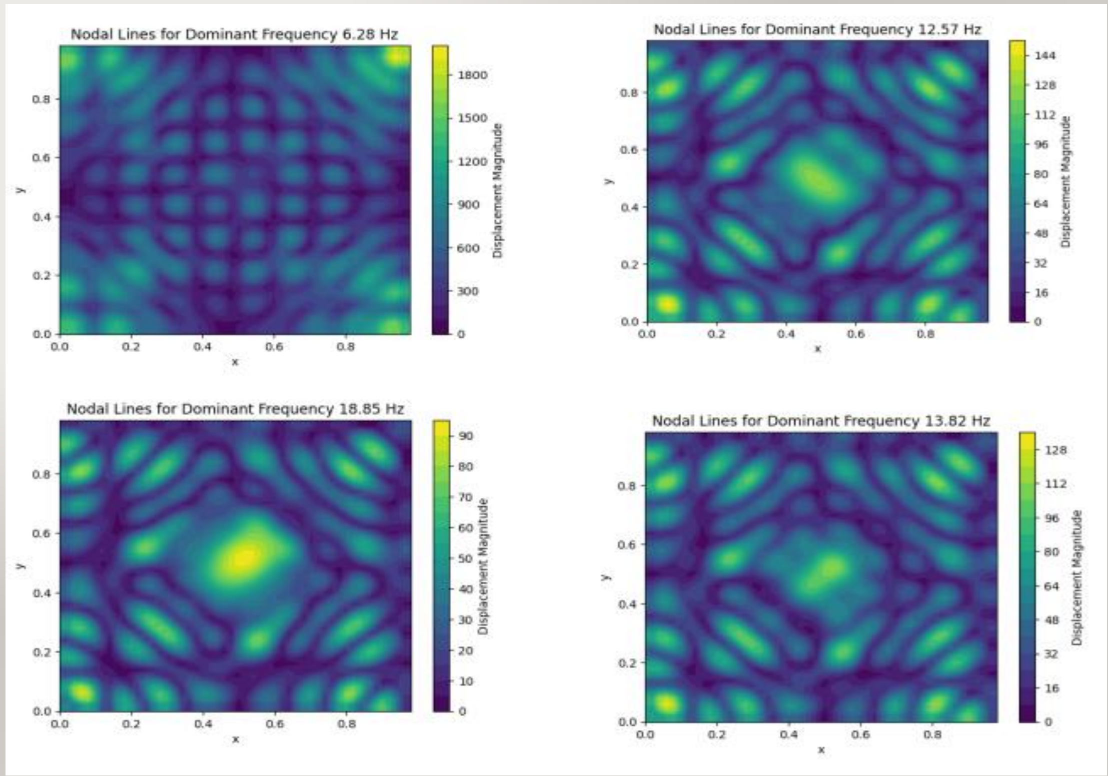

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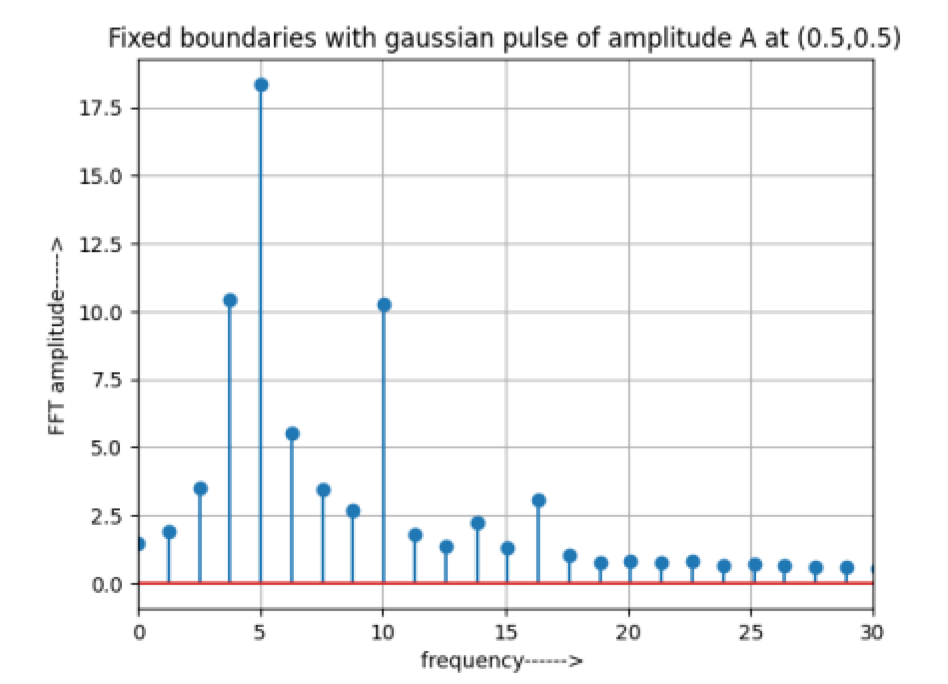

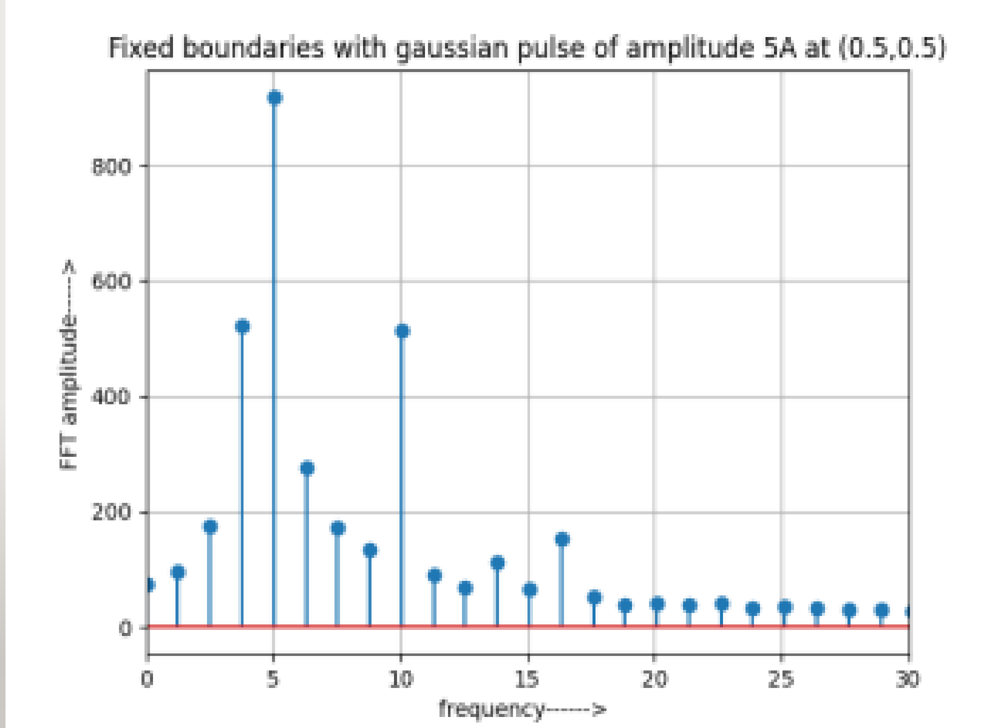

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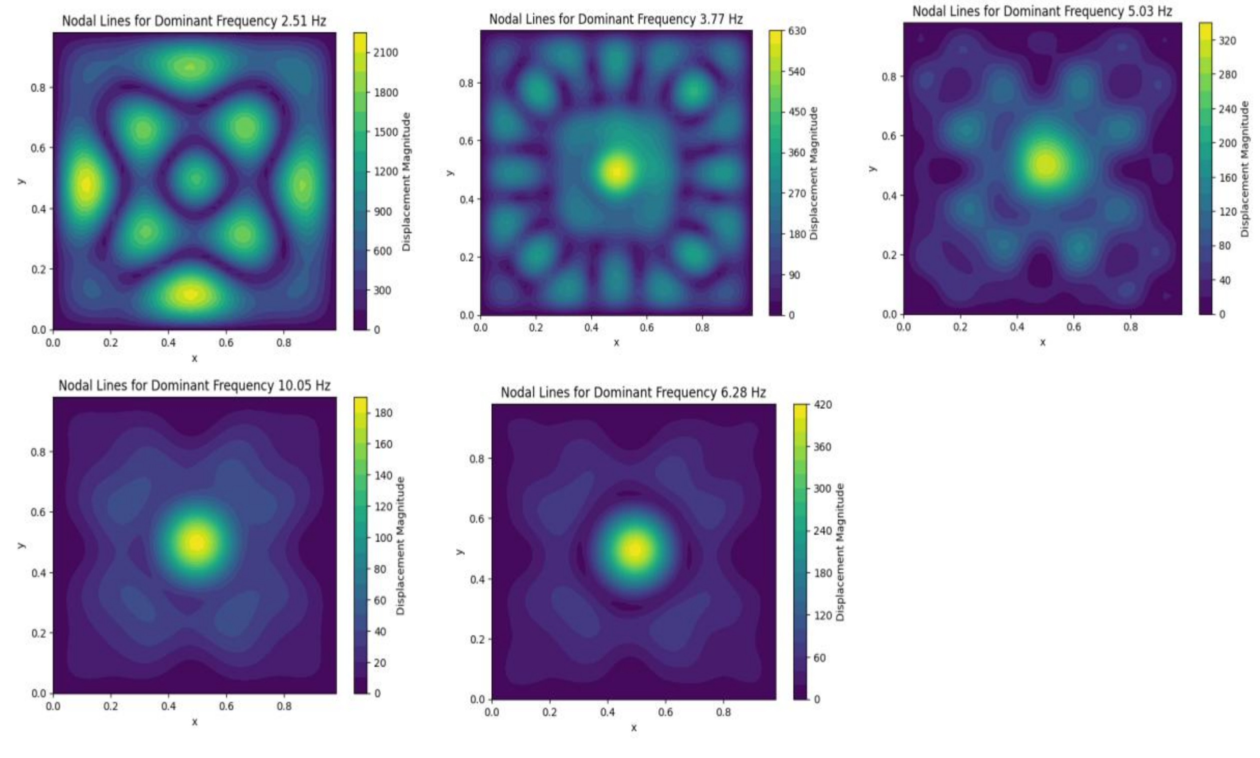

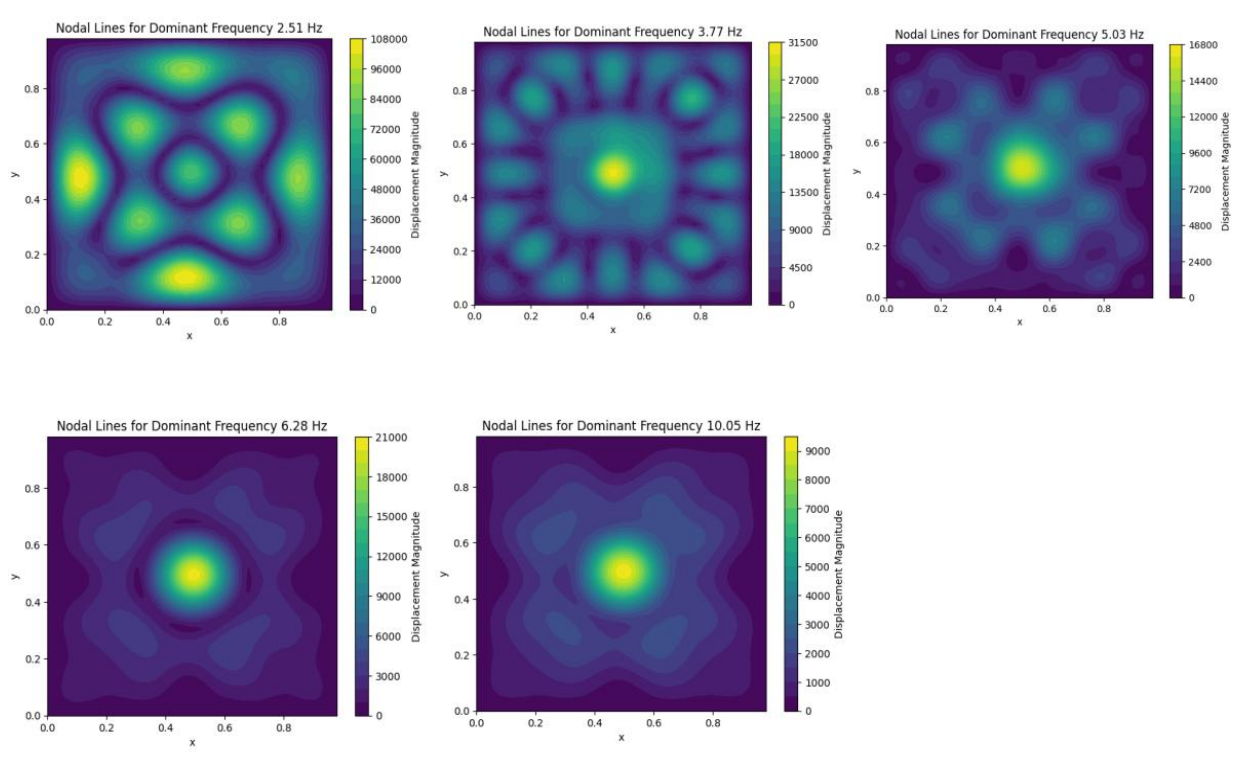

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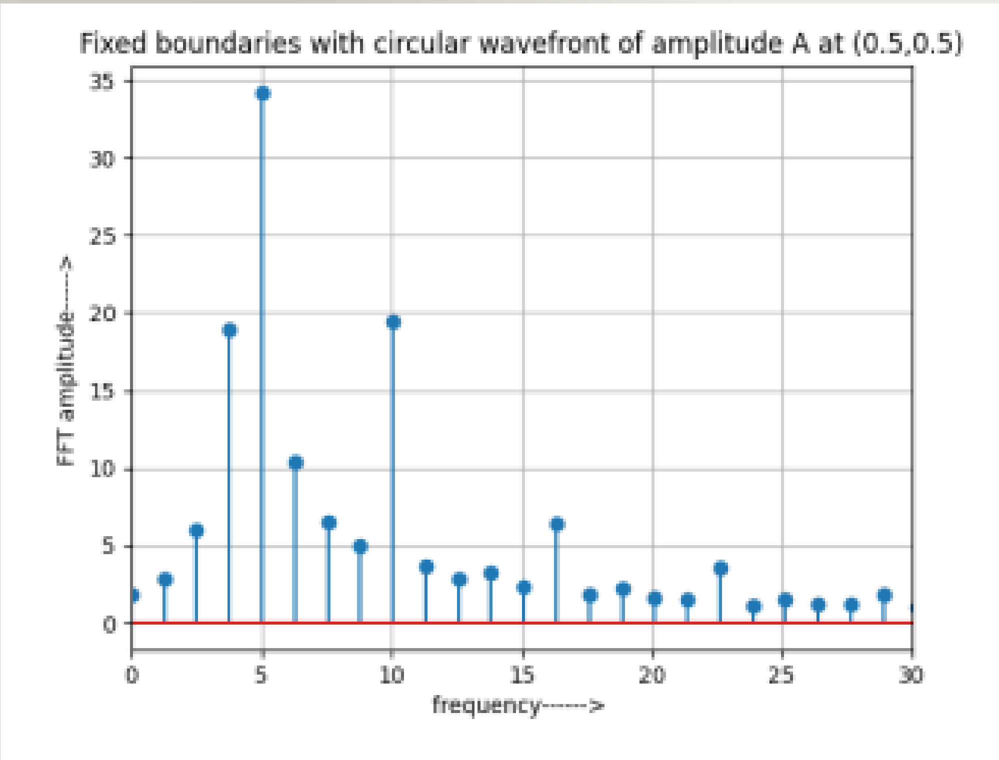

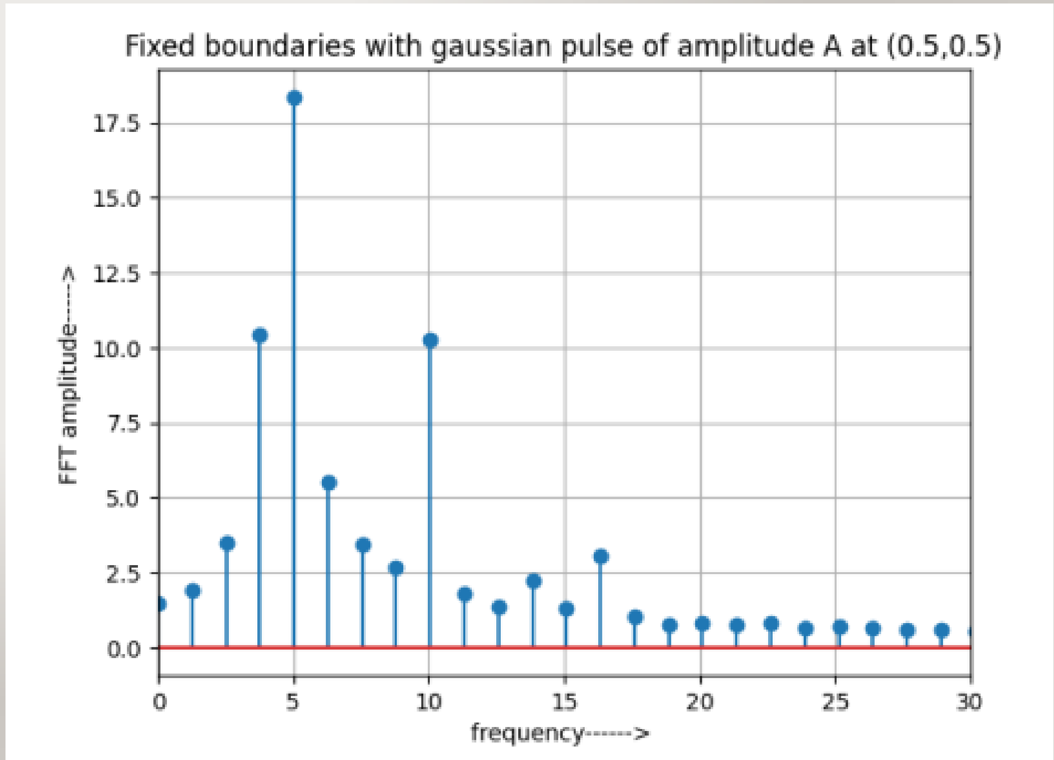

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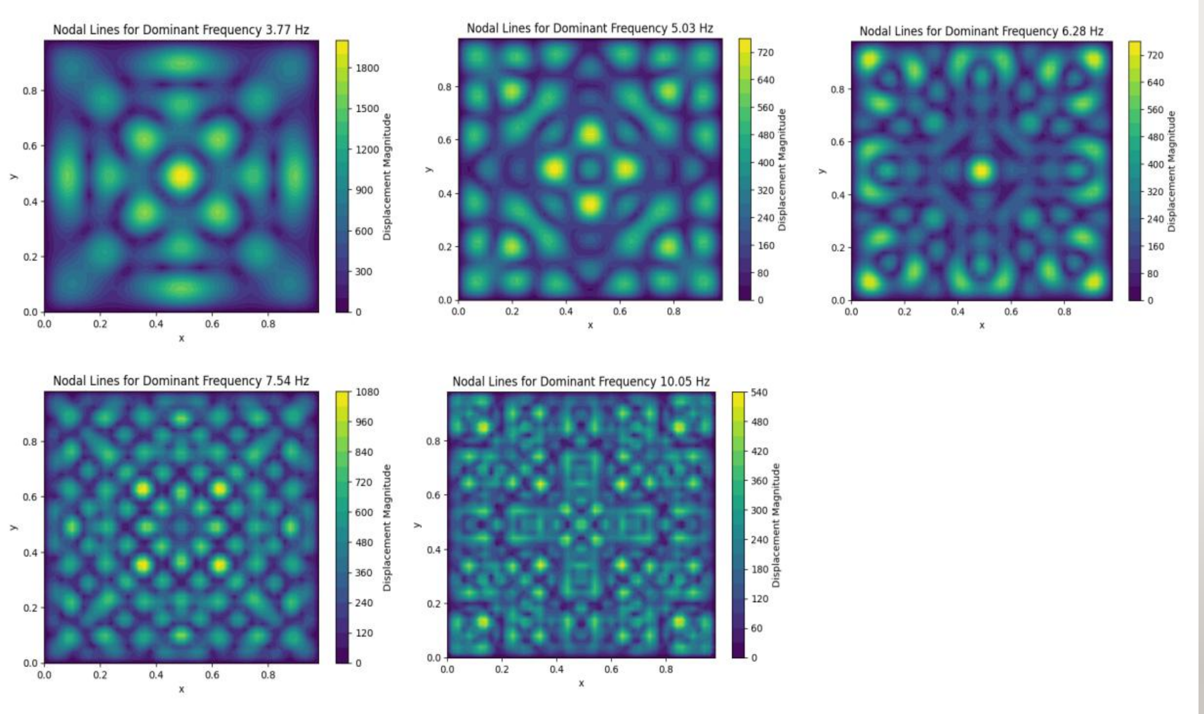
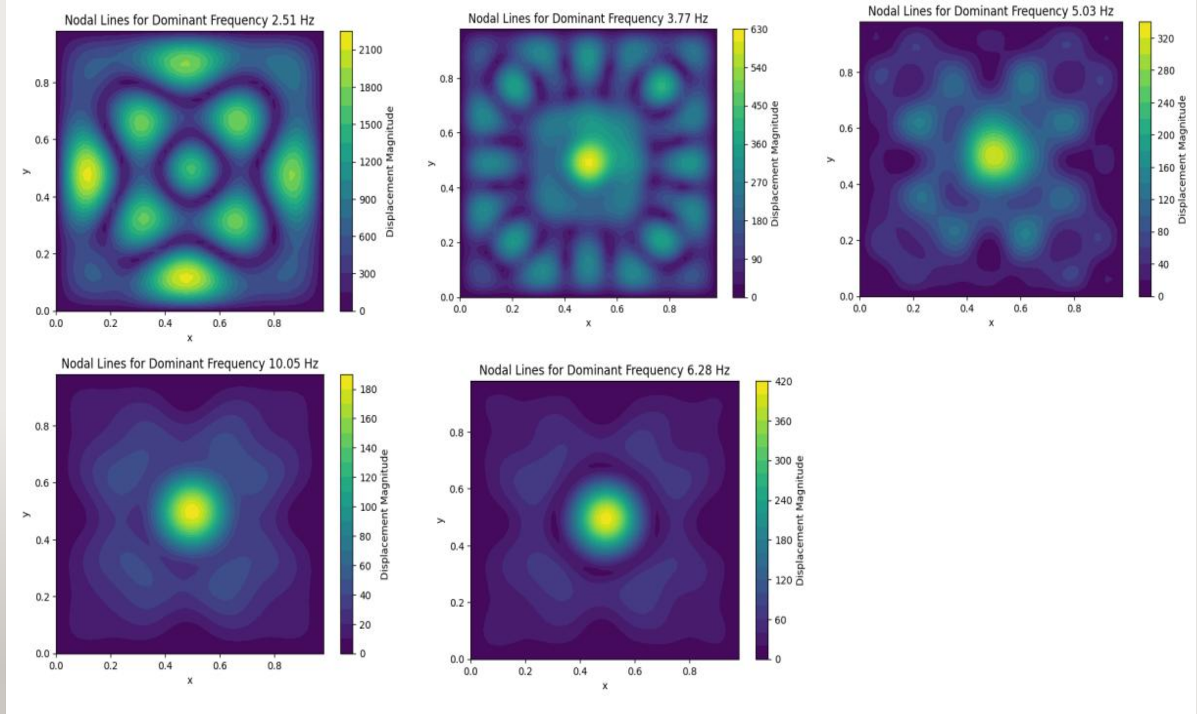
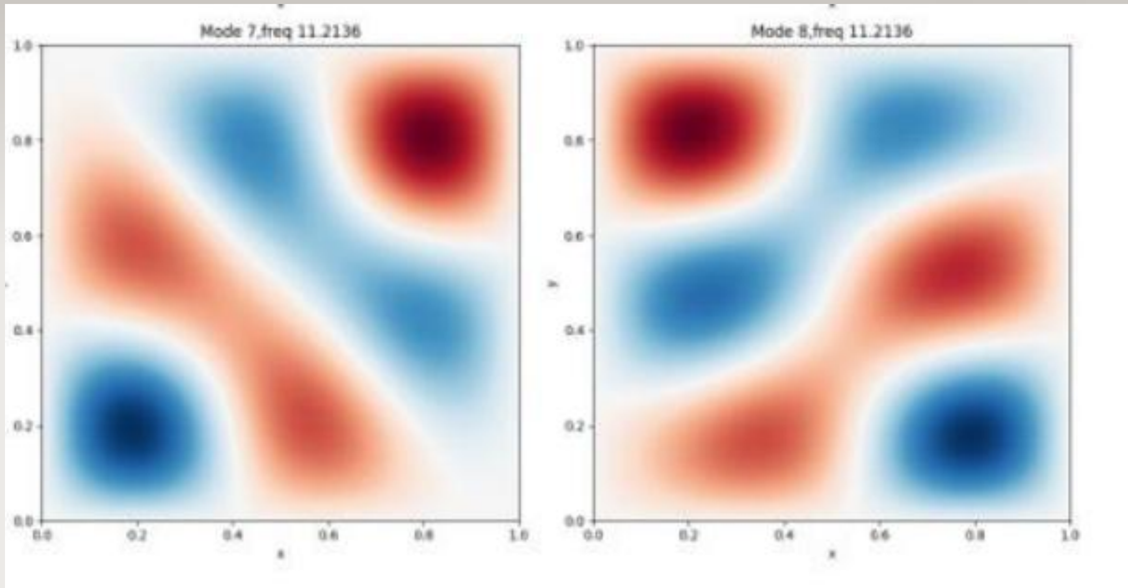
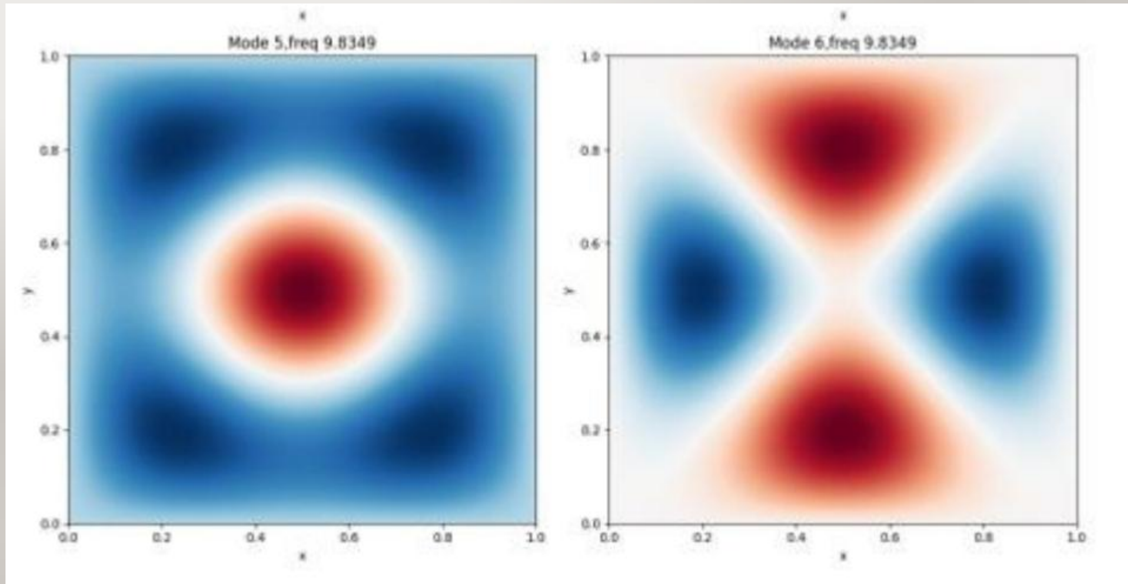




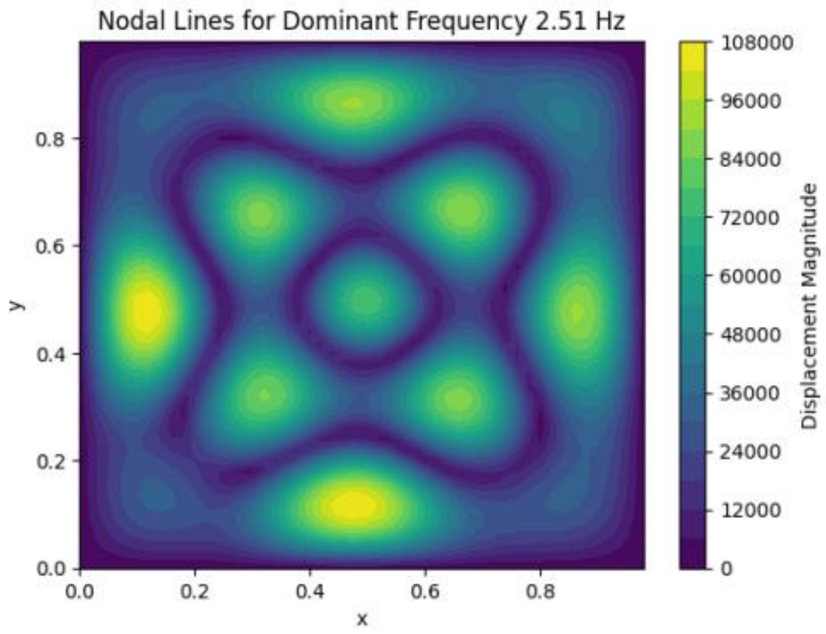
Degeneracy of modes

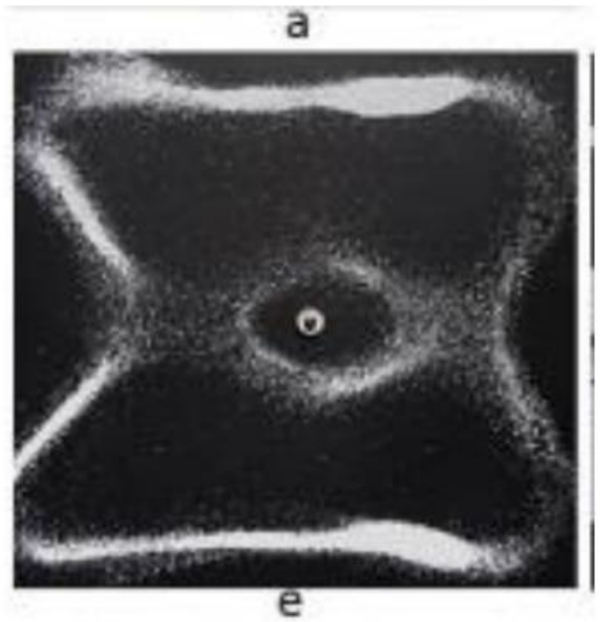
One of the fascinating phenomena of the waves on a 2d membrane is the degeneracy of the modes of vibration. Degeneracy means that for same value of nodal frequency there are different modes of vibration present.





Validation of results





Reference:- RESPONSE VARIATION OF CHLADNI PATTERNS ON VIBRATING ELASTIC PLATE UNDER ELECTRO-MECHANICAL OSCILLATION
https://www.ajol.info/index.php/njt/article/view/191724

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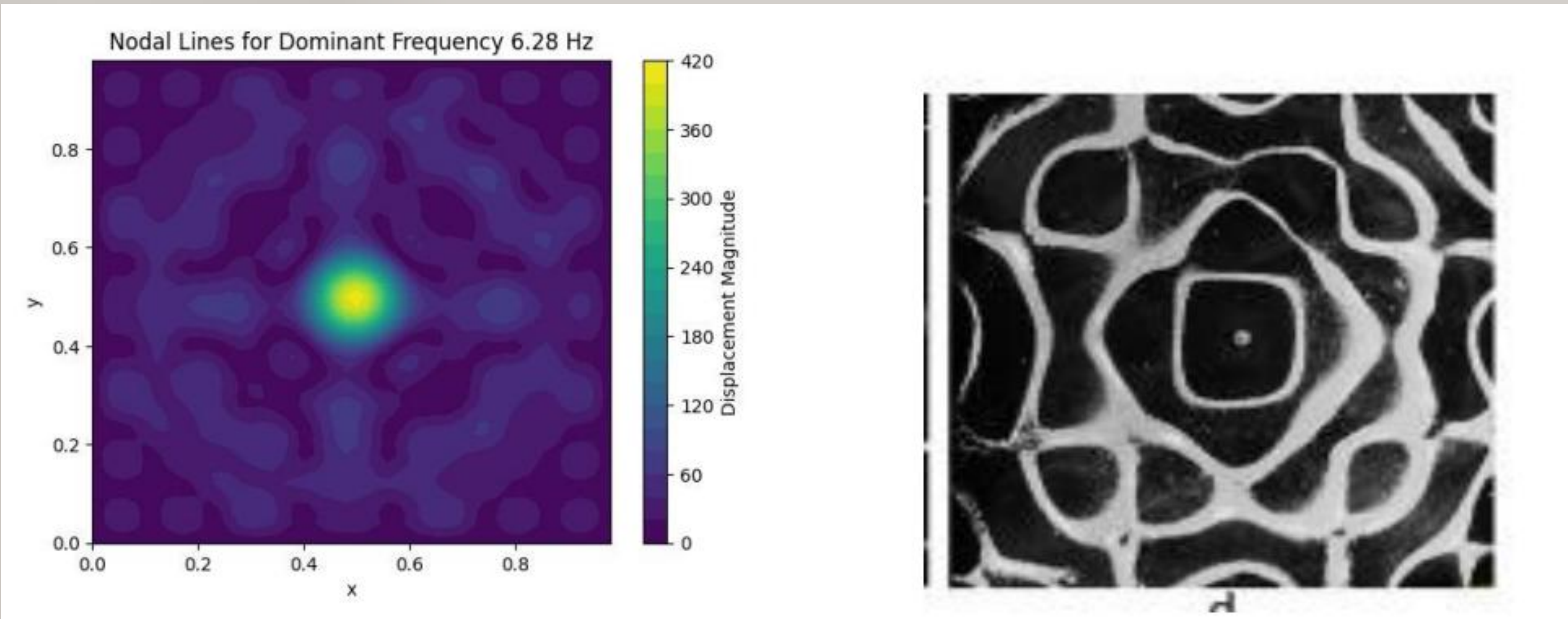

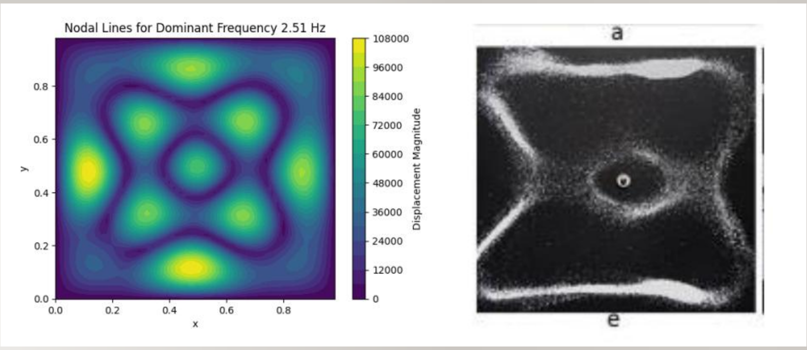

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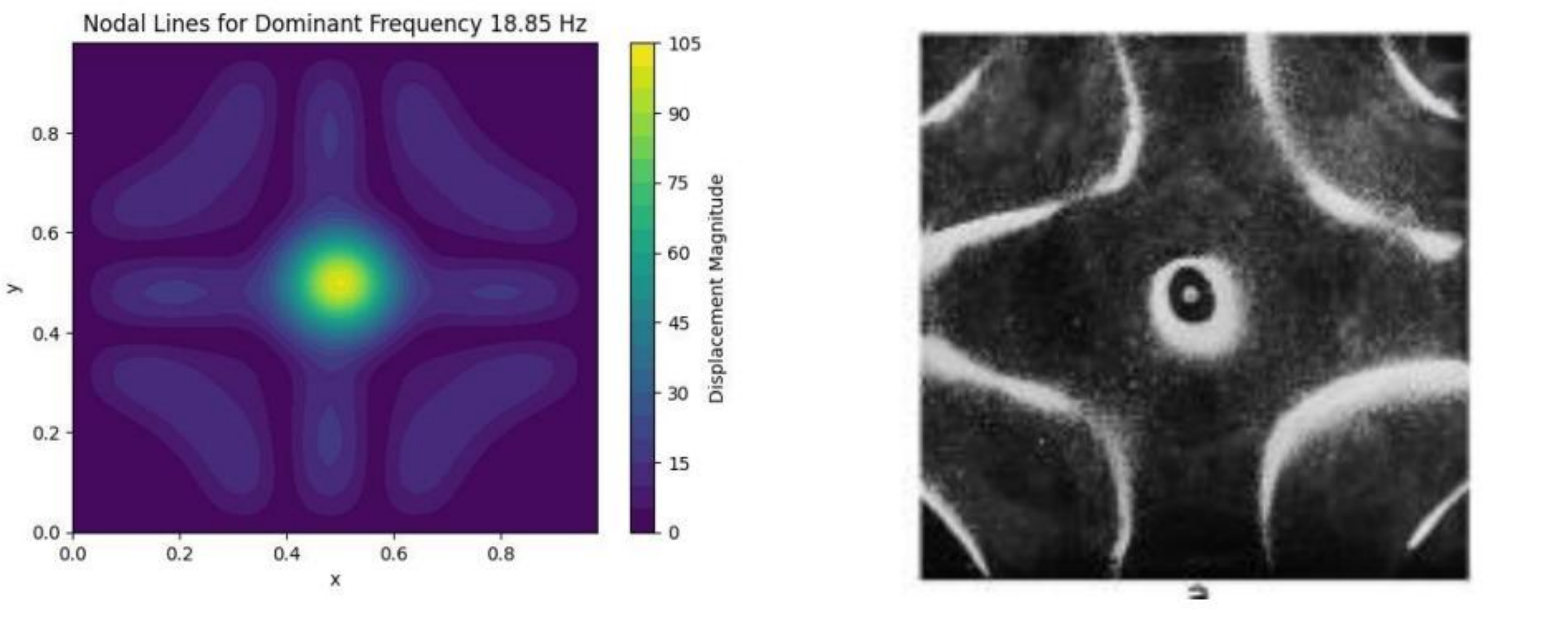

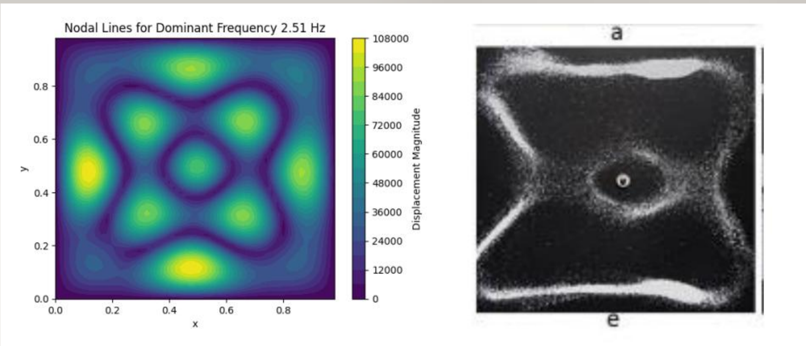

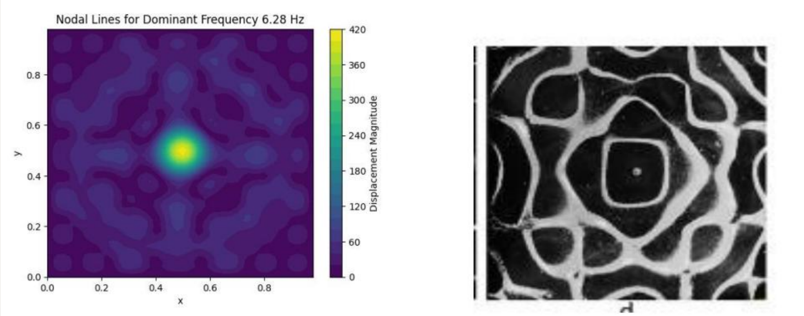

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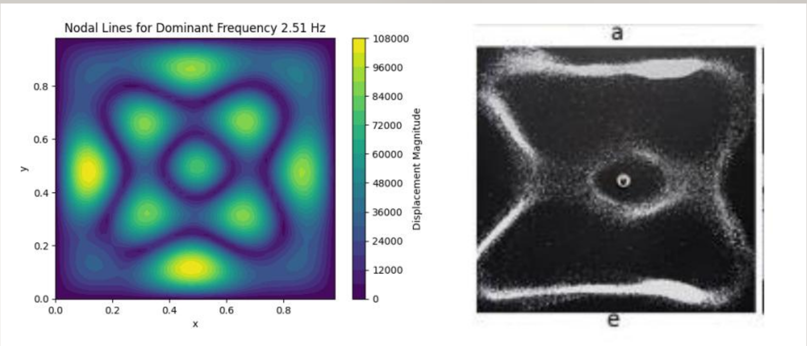

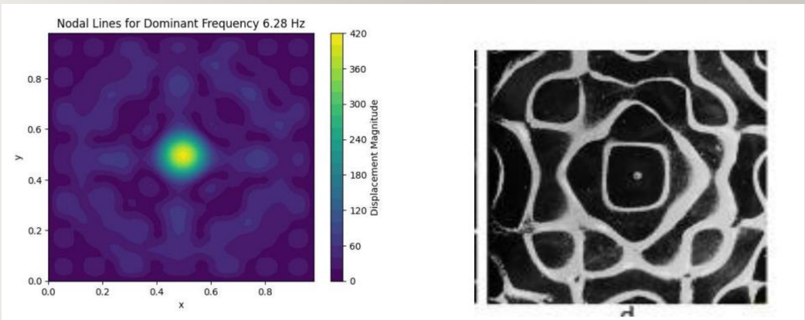

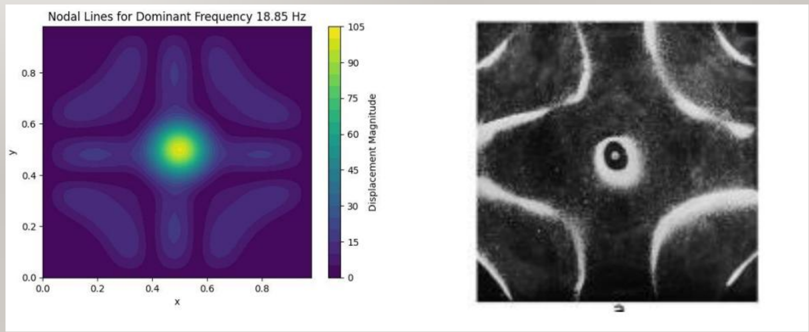

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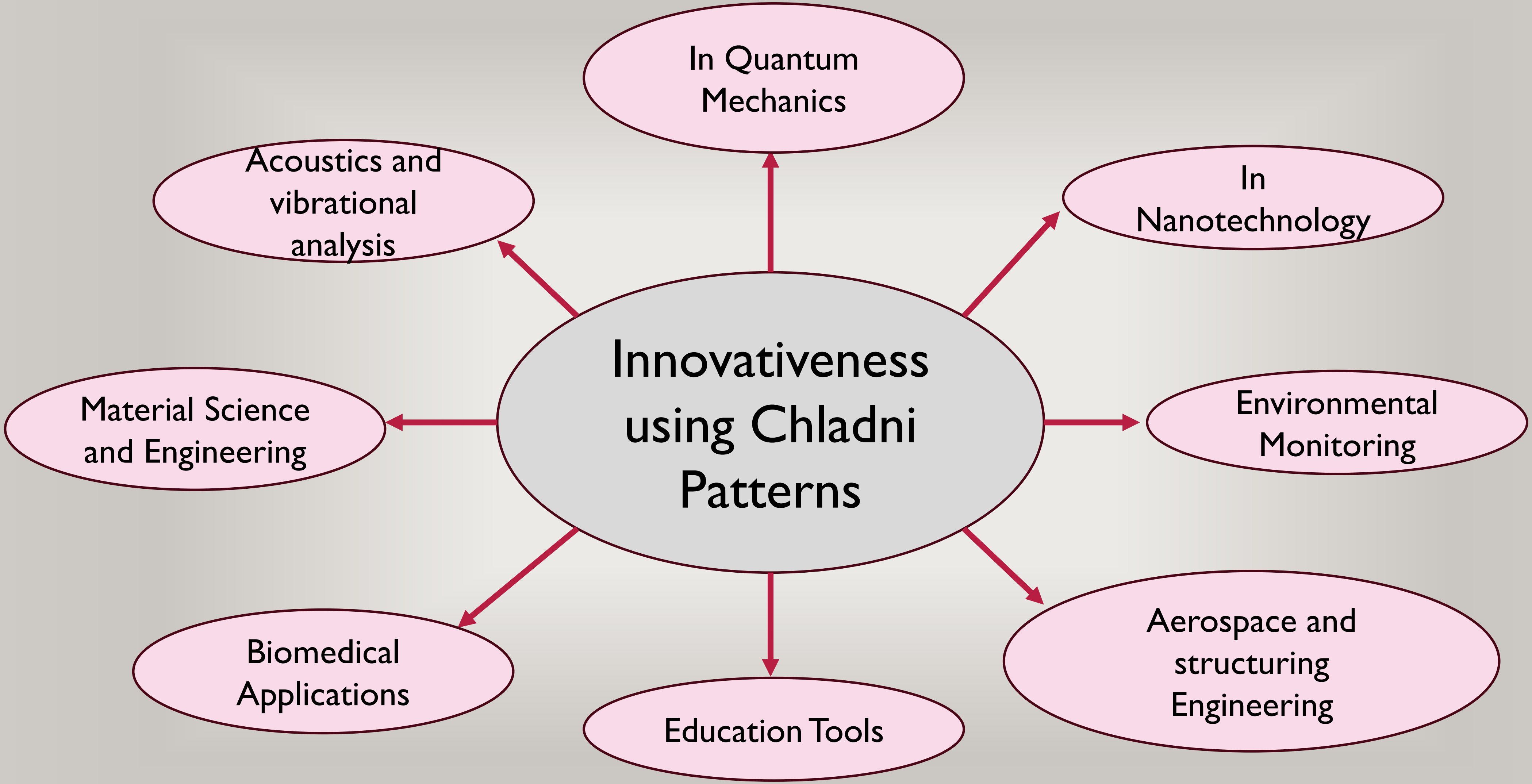
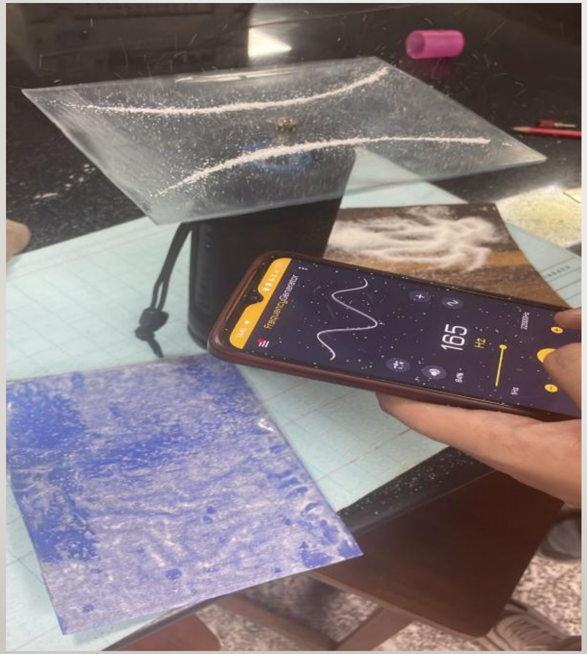

Innovativeness using Chladni Patterns
In Quantum Mechanics
In Nanotechnology
Environmental Monitoring
Aerospace and structuring Engineering
Education Tools
Biomedical Applications
Material Science and Engineering
Acoustics and vibrational analysis

Future work

Experimental setup to obtain Chladni patterns : Initial stage



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- An Introduction to Fourier Analysis | Russell L. Herman | Taylor & Fra (taylorfrancis.com) https://www.taylorfrancis.com/books/mono/10.1201/9781315367064/introduction-fourier-analysis-russell-herman
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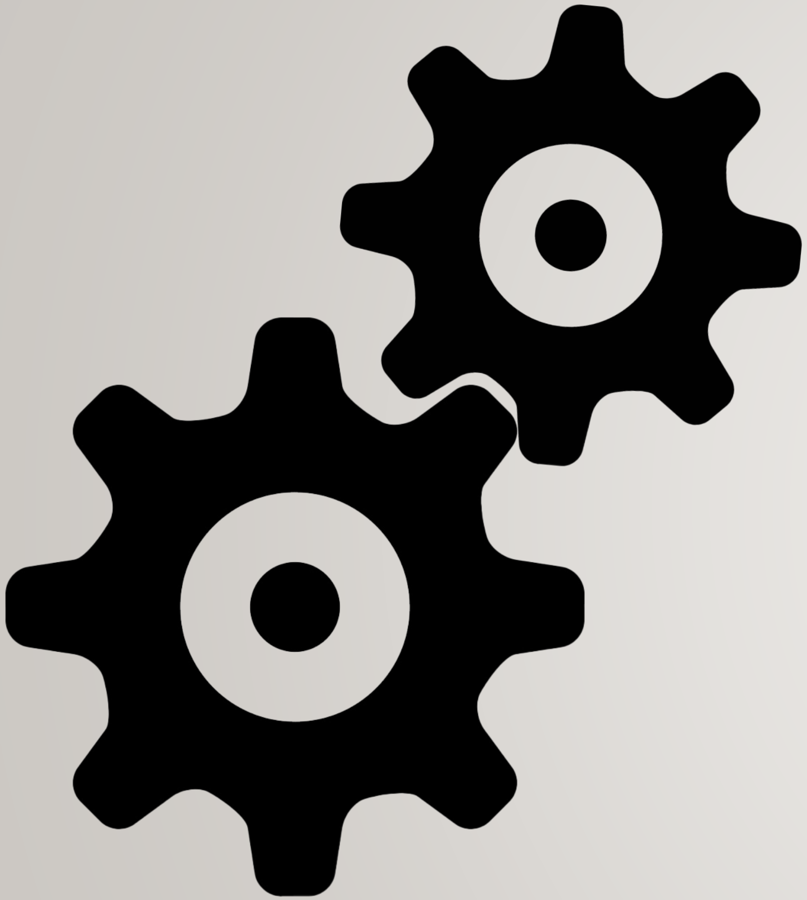
- A simple approach to determine the mode shapes of Chladni plates based on the optical lever method https://iopscience.iop.org/article/10.1088/1361-6404/ab2e2b
- How the Fourier Series Works By Mark Newman
- Fourier Analysis: An Introduction By Elias M. Stein

TABLE OF CONTENT



THANK YOU

Acknowledgments

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