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SYMPLECTIC AND NEURAL NETWORK APPROACHES TO PERIODIC ORBITS IN THE PLANAR THREE-BODY PROBLEM

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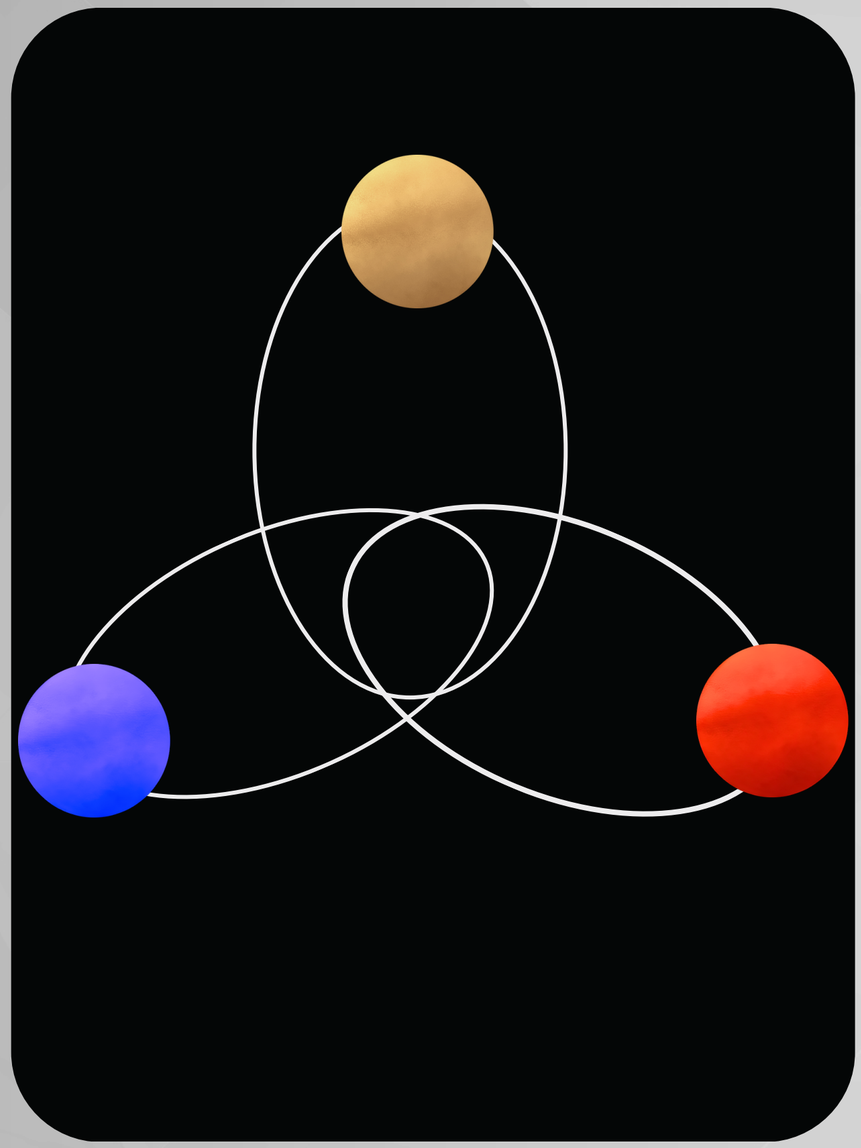

Navigating this Presentation

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TOPICS COVERED

01 Objectives

02 Motivation

03 Project Environment & Tooling

04 Methodology and Execution

05 Observations

06 Onward: Application & Outcomes

OBJECTIVES

- The planar three-body problem is a well-known classical challenge in celestial mechanics. It entails predicting the motion of three masses interacting under mutual gravitational attraction confined to a two-dimensional plane.

- The study aims to identify and analyze the various periodic solutions to this problem by leveraging numerical methods and Machine Learning algorithms.

- It also explores the consequences of adjusting the initial parameters (mass,initial conditions, integration time ,step size) on orbital behavior, thereby shedding light on the interplay amongst these parameters.

Motivation

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- In physics, the quest to describe natural systems through laws and expressions often meets the challenge of inherent complexity.The planar three-body problem incorporate this complexity, revealing the limitations of traditional analytical methods.
- While classical mechanics offers insights into two-body interactions, the three-body scenario presents a chaotic system that defies simple solutions
- Our research not only delves into the physical dynamics of this problem but also integrates advanced computational techniques of symplectic integrators and Neural Networks(NNs) to solve and comprehend these complex interactions
- As undergraduate researchers, we strive to deepen our understanding of the planar three-body problem to map out the fascinating patterns and behaviors that arise in this system.

Project Environment and Tooling

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Python (Language)

Python is a high-level, interpreted, and general-purpose dynamic programming language.

SPYDER

Spyder is a free and open source scientific environment designed for scientists, engineers and data analysts.

COLAB

Colab is a free cloud service that provides a platform to create and share interactive notebooks with code,extensively used for ML computation.

NUMPY

Used for generating arrays, and performing numerical computations

PANDAS

Used for creating dataframe of trajectory points.

MATPLOTLIB

For plotting the orbit trajectories

PYTORCH

PyTorch is a flexible and powerful framework for building and deploying ML Models

Methodology and Execution-

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Model Formulation by defining equations of motion for 3 bodies using Newtonian Approach

- **Theoretical Approaches**
  - Exploring stable orbits , sets of isosceles collinear configurations
    - Phase space, Zero Collision,No momentum
- **Numerical Methods**
  - Symplectic Integrator- Leapfrog Algorithm
  - Plotting trajectories with different initial conditions
    - Analyzing orbits by changing position,velocity and time coordinates
  - Inspecting stability with Phase Space diagrams
- **Machine Learning Algorithm(ANNs)**
  - Neural Network Predictions

OBSERVATIONS OVERVIEW-

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Figure-8 orbit

Periodic and more stable orbit

Butterfly Orbit

Aperiodic and chaotic in nature

Moth Orbit

Aperiodic and chaotic in nature

Hierarchical Orbits

Special class of orbits resembling to three star system

Machine Learning Algorithm

Problem Solving with ANNs approach

RESULTS & DISCUSSION

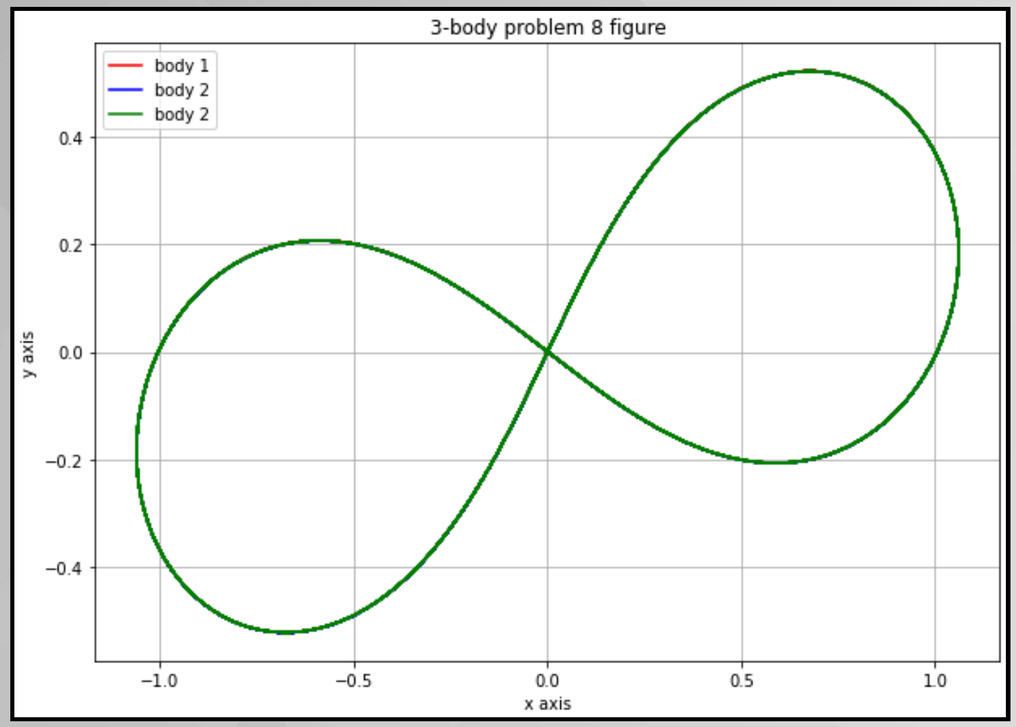


Fig 1- Figure 8 Orbit

- The orbit produces stable nature with integration time of 1000s,resulting periodic orbit.

Initial Conditions-

- Masses- m1=m2=m3=1 ; G=1
- Positions(x,y)- r1=[-1.,0] , r2= -r1 , r3= [0,0]
- Velocity (v1x,v1y) = (0.3489,0.5306)=v2, v3= -2*v1

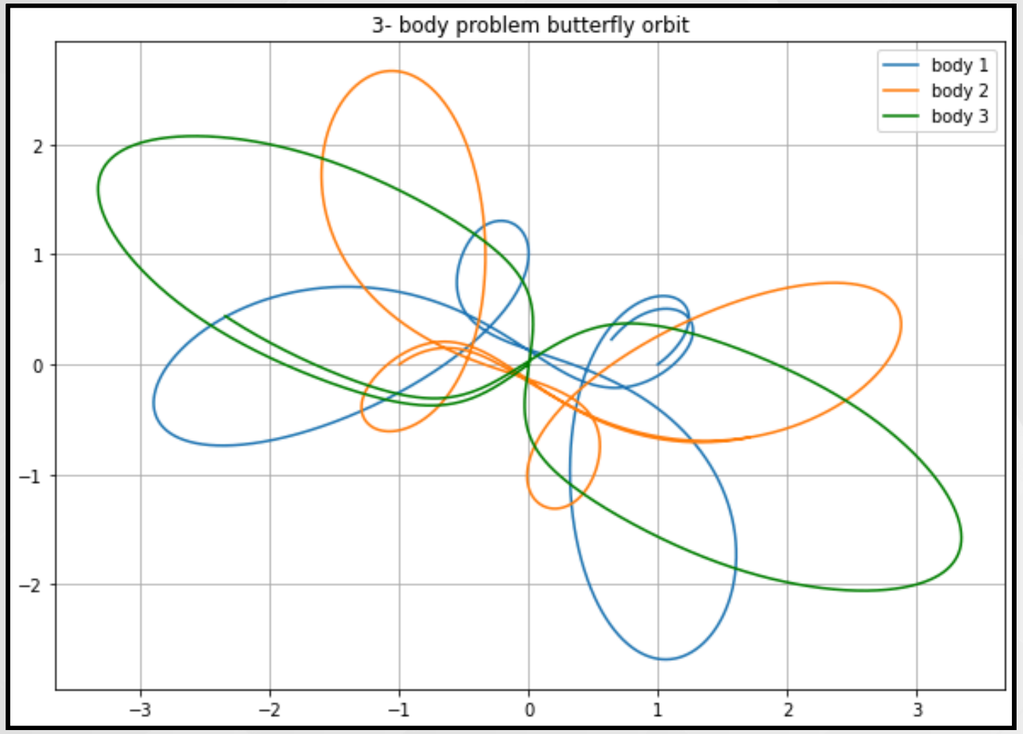


Fig 2- Butterfly Orbit

- The orbit produces chaotic nature over with integration time of 43 seconds,resulting aperiodic orbit.

Initial Conditions-

- Masses- m1=m2=m3=1 ; G=1
- Positions(x,y)- r1=[-1.,0] , r2= -r1 , r3= [0,0]
- Velocity (v1x,v1y) = (0.6150,0.5226)=v2,v3= -2*v1

RESULTS & DISCUSSION

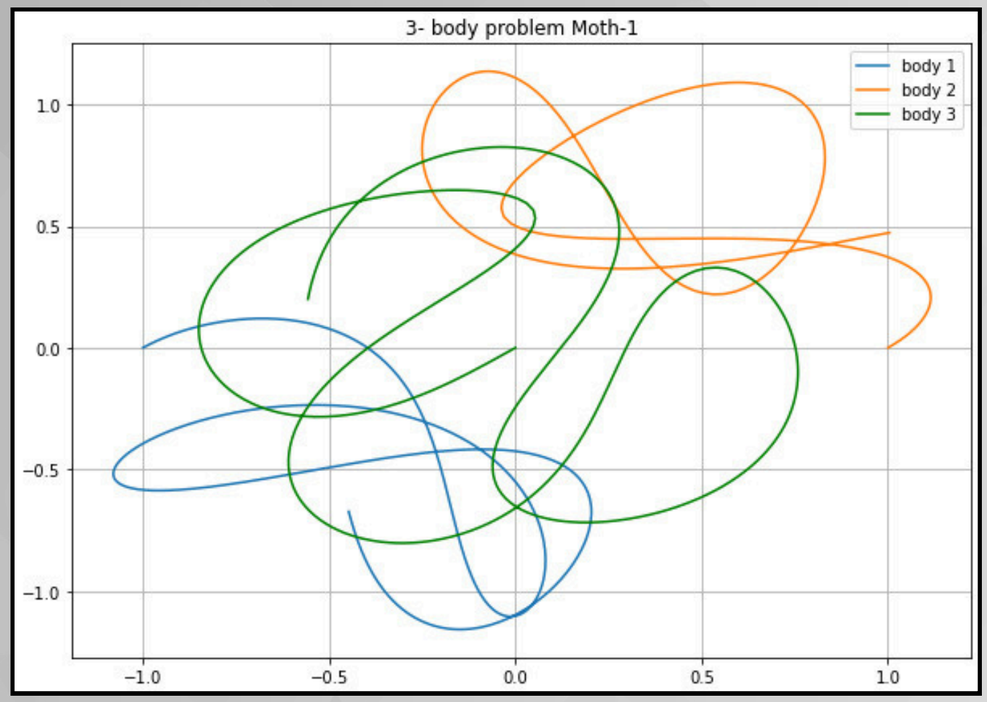


Fig 1- Moth 1 Orbit

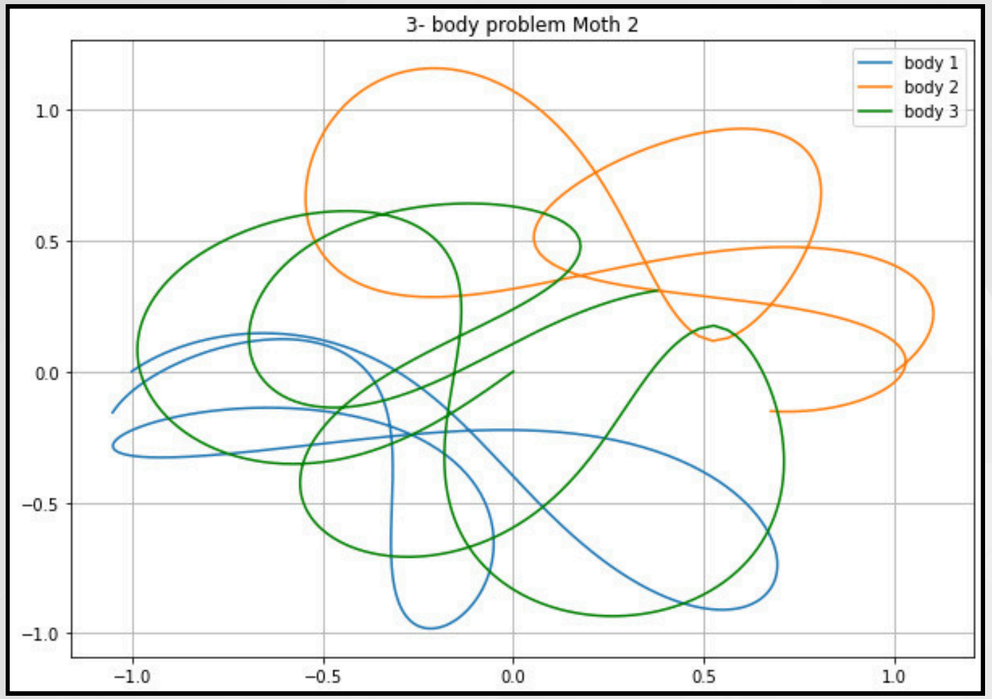
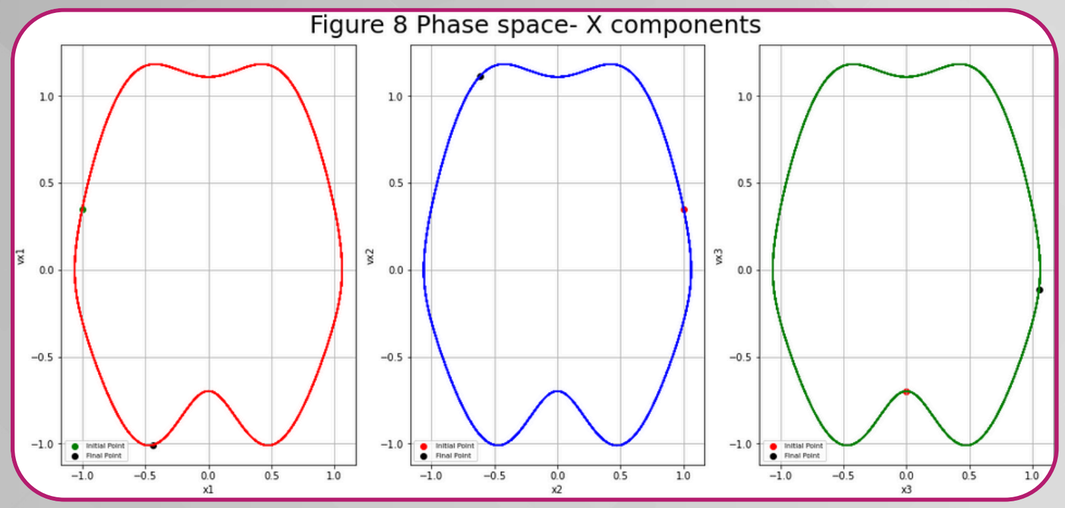


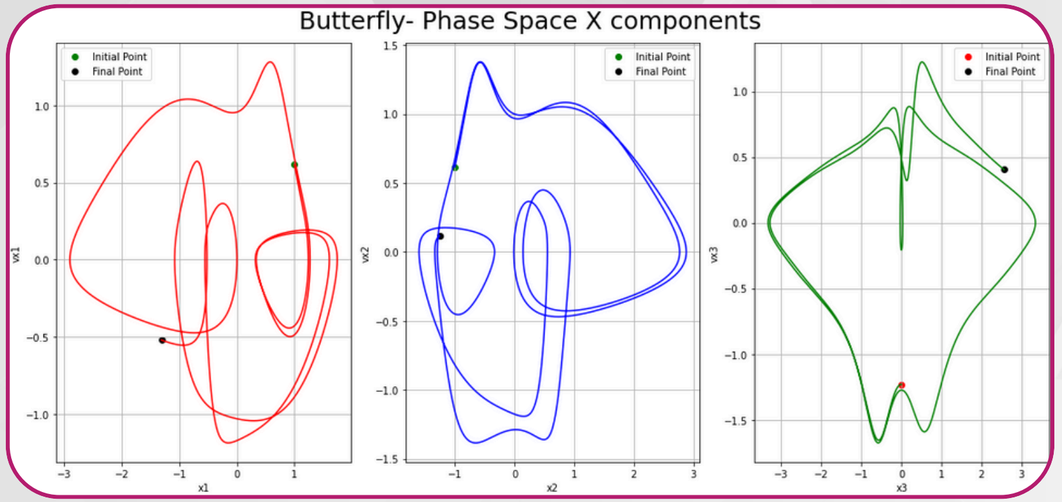
Fig 2- Moth-2 Orbit

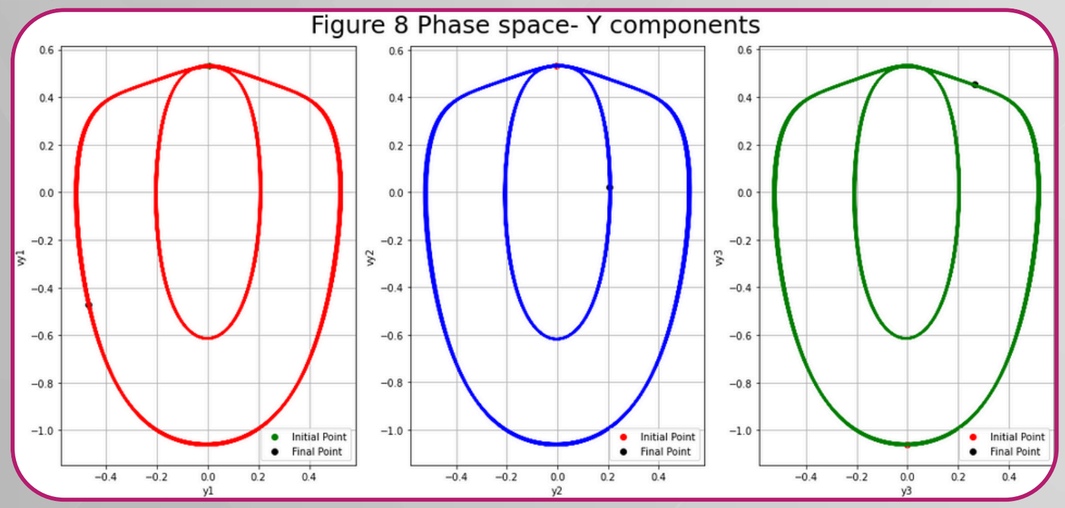
- Both Moth 1 and Moth 2 orbits exhibit a highly chaotic nature from the beginning of the integration time.
- The two orbits have similar initial condition but different time steps. This goes on to show the high sensitivity of the orbits to the computational parameters of the problem.

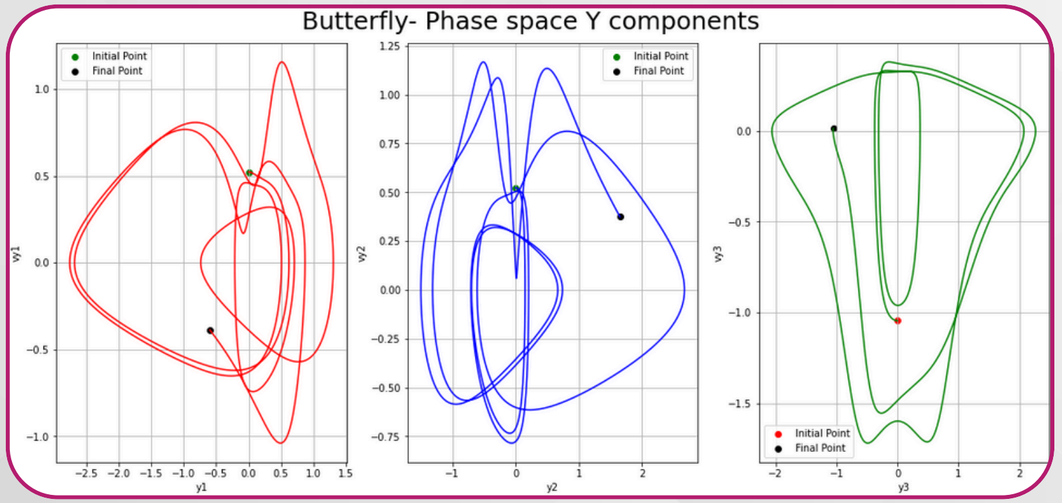
Understanding Stability Through Phase Space

Phase Space - A phase space of a dynamical system is a theoretical space where every state of the system is mapped to a unique spatial location.The graphs that result from plotting trajectories in phase space are recalled as **Phase Portraits.**

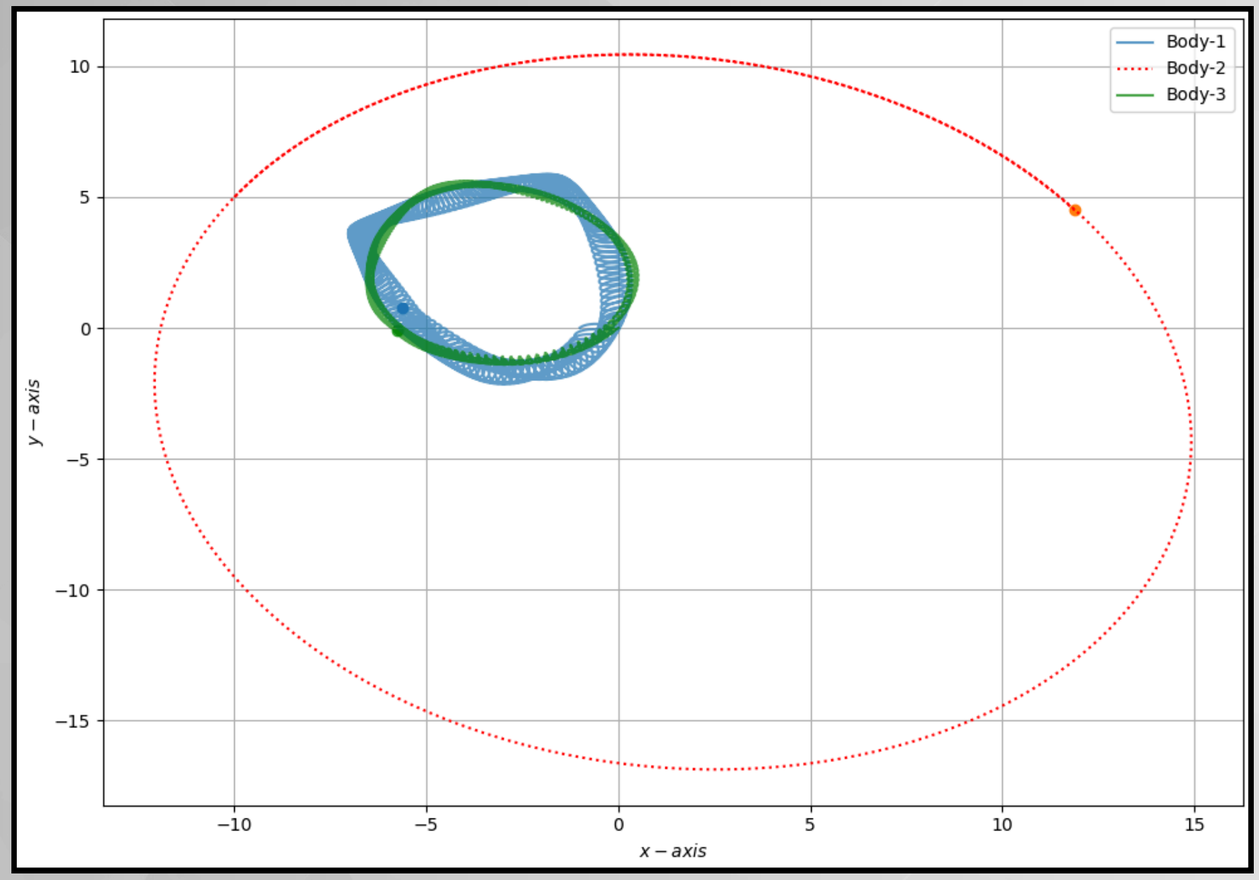


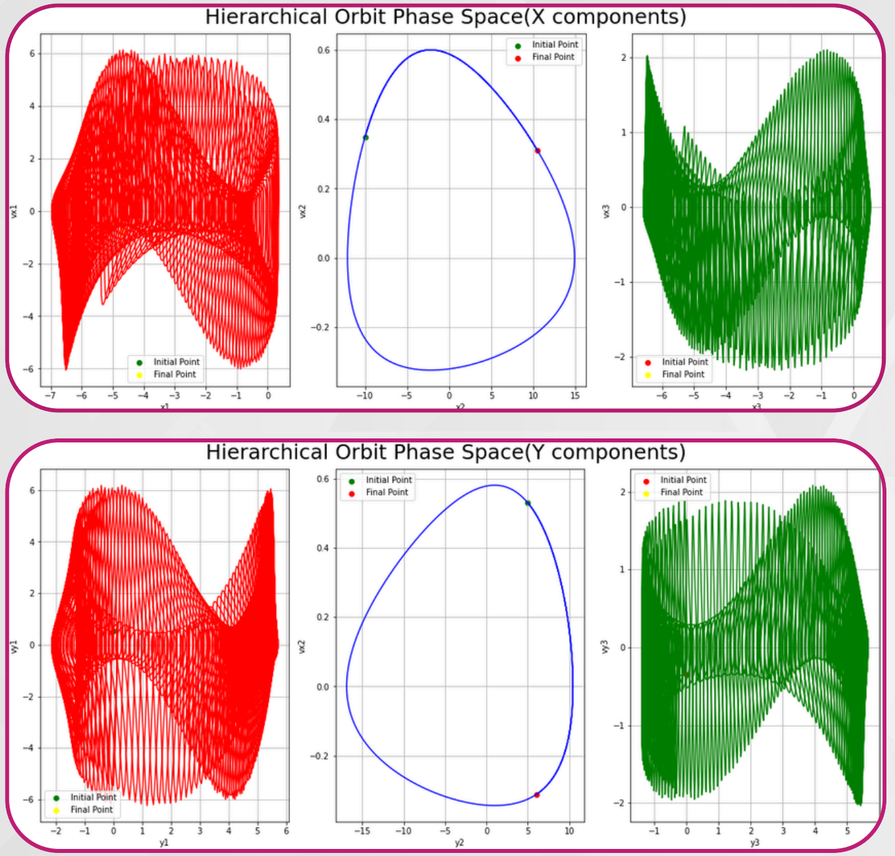






SPECIAL CASE- HIERARCHICAL ORBIT





- Examples- Triple star system
- Nature - The hierarchical structure ensures that there is a significant separation of orbital scales and tend to be more stable with change in integration time.

Initial Conditions-

- Position- (x1,x2,x1',x2') =(−1,0,−10,5)
- Velocity= (v1,v2) = (0.3489,0.5306)
- Masses- m1=m2=1, m3= 3 ; G=1

Fig- Phase Portrait of Heirarchical Orbit

CODING OVERVIEW

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1. *Importing Libraries and Defining Parameters:*

```
import numpy as np
import matplotlib.pyplot as plt
G=1
dt=0.1
t=np.arange(0,100+dt,dt)
nt=len(t)
m1,m2,m3=1,1,1
r1,r2,r3=np.zeros((nt,2)),np.zeros((nt,2)),np.zeros((nt,2))
v1,v2,v3=np.zeros((nt,2)),np.zeros((nt,2)),np.zeros((nt,2))
```

2. *Transforming the Equations of Motion:*

```
def f1(r,m2,m3,j):
    return -G*(m2*(r[0][j]-r[1][j])/(np.linalg.norm((r[0][j]-r[1][j])))**3
               +m3*(r[0][j]-r[2][j])/(np.linalg.norm((r[0][j]-r[2][j])))**3))
def f2(r,m1,m3,j):
    return -G*(m1*(r[1][j]-r[0][j])/(np.linalg.norm((r[1][j]-r[0][j])))**3
               +m3*(r[1][j]-r[2][j])/(np.linalg.norm((r[1][j]-r[2][j])))**3))
def f3(r,m1,m2,j):
    return -G*(m1*(r[2][j]-r[0][j])/(np.linalg.norm((r[2][j]-r[0][j])))**3
               +m2*(r[2][j]-r[1][j])/(np.linalg.norm((r[2][j]-r[1][j])))**3))
```

3. *Building the Leapfrog Algorithm:*

```
def lfrog(r,v):
    for i in range(len(t)-1):
        r[0][i+1]=r[0][i]+v[0][i]*dt+f1(r,m2,m3,i)*0.5*dt**2
        r[1][i+1]=r[1][i]+v[1][i]*dt+f2(r,m1,m3,i)*0.5*dt**2
        r[2][i+1]=r[2][i]+v[2][i]*dt+f3(r,m1,m2,i)*0.5*dt**2
        v[0][i+1]=v[0][i]+0.5*dt*(f1(r,m2,m3,i)+f1(r,m2,m3,i+1))
        v[1][i+1]=v[1][i]+0.5*dt*(f2(r,m1,m3,i)+f2(r,m1,m3,i+1))
        v[2][i+1]=v[2][i]+0.5*dt*(f3(r,m1,m2,i)+f3(r,m1,m2,i+1))
    return r,v
```

4. *Initializing the Conditions:*

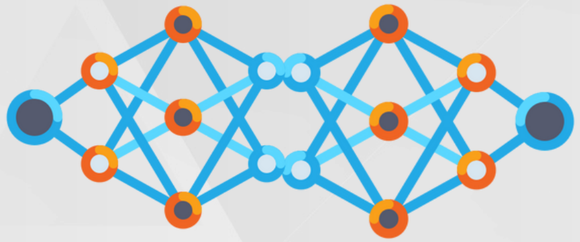
- *Figure of 8 Orbit*

```
r1[0]=[-1.00,0.00]
v1[0]=[0.3489,0.5306]
r2[0]=-r1[0]
v2[0]=v1[0]
v3[0]=-2*v1[0]
r=[r1,r2,r3];v=[v1,v2,v3]
r_new,v_new=lfrog(r,v)
```

5. *Plotting the Orbits:*

```
plt.plot(r_new[0][:,0],r_new[0][:,1],linestyle='--',label='Body 1')
plt.plot(r_new[1][:,0],r_new[1][:,1],linestyle=':',label="Body 2")
plt.plot(r_new[2][:,0],r_new[2][:,1],label='Body 3')
plt.grid()
plt.xlabel("x")
plt.ylabel("y")
plt.legend()
plt.show()
```

Physics Informed Neural Networks(PINNs & Artificial Neural Networks(ANNs)



PINNs is a new and exciting technique in scientific computing that combines the power of Artificial Neural Networks (ANN) with traditional physics knowledge.

What it is: A type of universal function approximator, meaning it can learn and represent complex relationships from data. Integrates knowledge of physical laws into the learning process, often represented by PDEs & ODEs. Used to solve complex problems in engineering & physics.

Advantages of PINNs:- 1- Mesh Free Method.
2- Inverse Problem Solving
3- Avoidance of Convergence Challenges

ANN is based on the idea of optimizing the network's parameters and hyperparameters to minimize the difference between the predicted output and the actual target values in a given dataset.

Gradient-based methods such as backpropagation are usually used to estimate the parameters of the network. During the training phase, ANNs learn from labeled training data by updating their parameters (weights and biases) in order to minimize a defined loss function.

Neural Network Predictions-

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Observation

- By modeling and training the network, the losses were minimised and the hyper-parameters like number of neurons, neural layers, learning rate, etc. were optimised.

- Initially, the trajectories along the individual x-and y-directions were optimised following which the orbit was predicted.

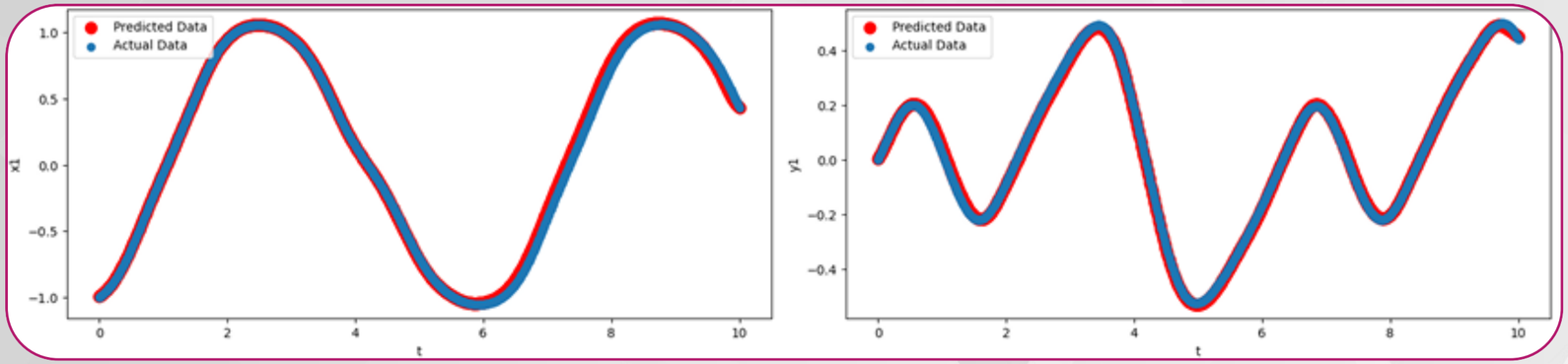

Figure 1 - Errors in prediction of x and y trajectory

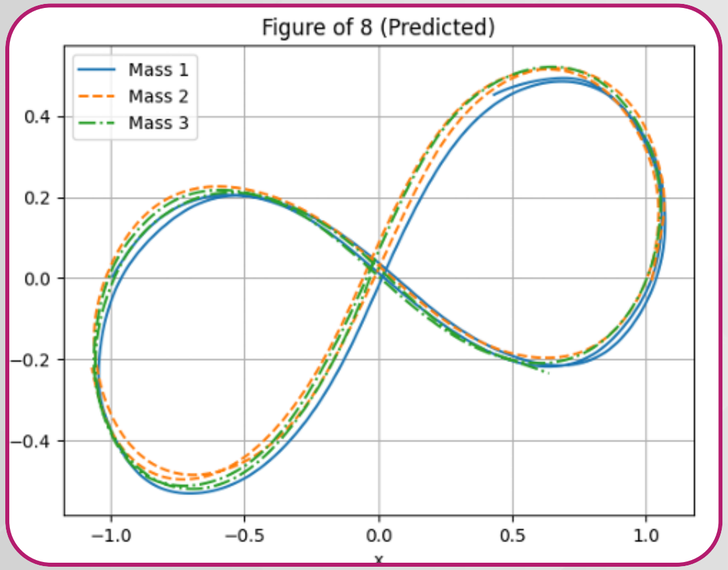

Figure 2- Predicited Orbit

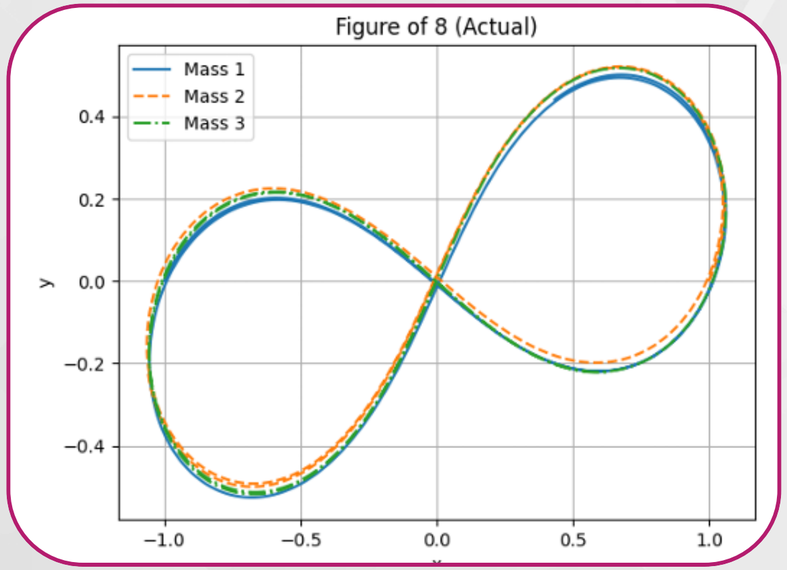

Figure 3- Actual Orbit

Future Applications

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- *Astrophysics and Cosmology -* Studying the formation and evolution of star systems, galaxies, and planetary systems, including the dynamics of triple star systems and interactions between multiple galaxies.

- *Space Debris Management -* Predicting and mitigating collisions in space by modeling the interactions between various pieces of space debris and operational satellites.

- Lagrange Point Missions - Exploring and utilizing Lagrange points (positions in a three-body system where gravitational forces balance) for space telescopes, observatories, and space habitats.

- *Planetary Defense -* Predicting the trajectories of potentially hazardous asteroids and comets that may be influenced by the gravity of multiple bodies, improving our ability to prevent or mitigate impacts.

- *Quantum and Classical Chaotic Systems-* Using the three-body problem as a testbed for understanding chaotic behavior in classical and quantum systems, which has implications for fields ranging from material science to cryptography.

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THANK YOU

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